Pitch Tracking of a High Dynamic Platform Using LQR

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***Abstract* — The research focuses on designing a pitch control system for a MANPAD using appropriate guidance algorithm to intercept a target. State space models, corresponding to several time segments thorough the flight of the missile, were obtained and a Linear Quadratic Regulator (LQR) based pitch controller was designed for each time segment in order to track the desired pitch commands obtained from the trajectory.**

Keywords—pitch control; Liner Quadratic Regulator

I. NOMENCLATURE

I	Inertia Matrix
I_{xx}	Moment of Inertia about x axis
I_{yy}	Moment of Inertia about y axis
I_{zz}	Moment of Inertia about z axis
nc	Acceleration Command
N’	Effective Navigation Ratio
Vc	Missile-Target Closing Velocity
λ	Line of Sight
VM	Missile Velocity Magnitude
VT	Target Velocity Magnitude
nt	Target Acceleration
RTM	Missile Target Line of Sight Length
L	Lead Angle
HE	Heading Error
$\boldsymbol{\beta}$	Flight Path Angle of Target
θ	Pitch Angle

II. INTRODUCTION

The missile used for the purpose of this paper is a short range, surface to air, Man Portable Air Defense System (MANPADS). Such vehicles are usually capable of traveling a distance of 5 km at speeds around Mach 2. The average flight time of these vehicles are around 10 seconds before they self-destruct. It is very rare to find exact specifications of such vehicles so a design of a MANPADS was estimated and the paper was proceeded in accordance with this design.

Proportional Navigation (PN) Guidance Law was used to define the trajectory of the vehicle in accordance with heading errors and target maneuvers to give a path to the controller to follow for different scenarios. From these trajectories we obtain the pitch commands required by the vehicle. These are used as reference commands that the controller will track. The dynamics of the vehicle are modeled using state-space models which change throughout the flight of the vehicle corresponding to variation in mass properties (mass and inertia tensors), stability derivatives and the center of gravity over the burn duration of the vehicle and different height regimes. To cater for this the flight was broken down into several time segments depending on the time the vehicle takes to hit the target, with each segment spanning one second, and LQR based controllers were implemented for each section.

III. VEHICLE MODEL

The vehicle was modeled using the standard shape of the MANPADS missile and consists of seeker section, servo section, warhead and a power unit. Fig. 1, 2 and Table 1 shows the specifications of these subsections. The vehicle has four vertical canards placed at 90^0 angles to each other and four aft fins placed at 90^0 angles to each other with 45^0 angles between canards and fins. Fig. 3, 4, 5 shows the vehicle CAD model. Similarly, the propellant weights and burn rate are discussed in Power Unit section.

TABLE 1. DIMENSIONS & WEIGHTS OF MISSILE SECTIONS

Section	Length/mm	Weight/Kg
Seeker Section	210	0.9
Servo Section	142	0.92
Warhead	164	1.42
Power Unit	1010	7.52

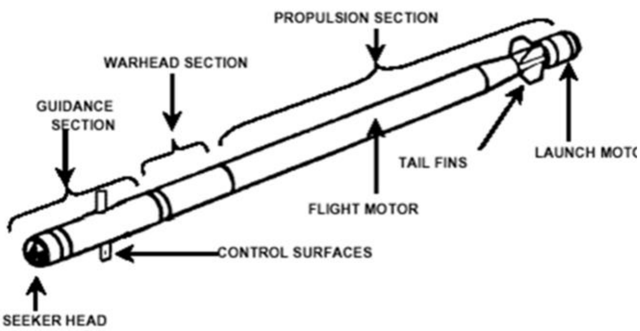

Fig. 1. Missile Sections

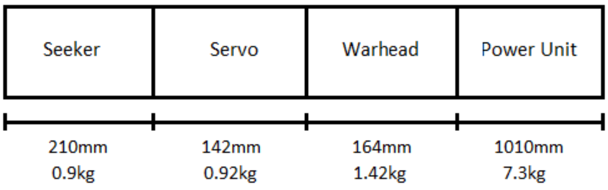

Fig. 2. Block diagram representation of missile parameters

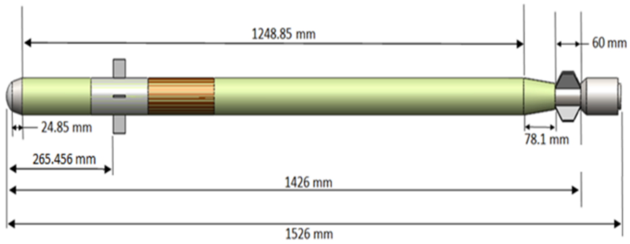

Fig. 3. Top view

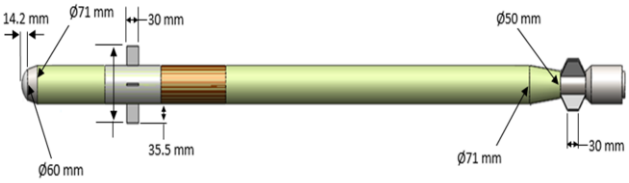

Fig. 4. Side view

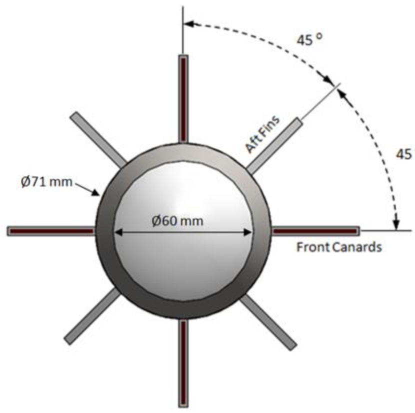

Fig. 5. Front view

To design a control system for the vehicle the first step is defining a mathematical model which simulates the behavior of the vehicle. The dynamics of the vehicle is in the form of equations of motion. The starting point for these equations are the standard linearized 6DOF equations of motion, which can be found in many books [1]

These can be divided into two parts:

1. Longitudinal motion.
2. Lateral motion.

For the purpose of our study we will use the longitudinal equations of motion. The standard linearized longitudinal equations of motion in state-space format, where the only control available is by deflecting the control surfaces, is given by (1), (2).

$$\dot{x} = Ax + BU \tag{1}$$

$$\begin{bmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} X_u & X_w & 0 & -g\cos\theta \\ Z_u & Z_w & U_e & -g\sin\theta \\ M_u^* & M_w^* & M_q^* & M_\theta^* \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} u \\ w \\ q \\ \theta \end{bmatrix} + \begin{bmatrix} X_{\delta e} \\ Z_{\delta e} \\ M_{\delta e}^* \\ 0 \end{bmatrix} [\delta e] \tag{2}$$

Where;

$$M_u^* = M_u + M_{\dot{w}} Z_u$$
$$M_u^* = M_w + M_{\dot{w}} Z_w$$
$$M_u^* = M_q + M_{\dot{w}} u$$
$$M_u^* = -M_u g \sin\theta$$
$$M_{\delta e}^* = Z_{\delta e} + M_{\dot{w}} Z_{\delta e}$$

Detail on how to get values of these variables is discussed in Stability Derivatives section.

V. Power Unit

The vehicle is powered by a power unit that consists of two parts:

1. Launch Motor
2. Flight Motor

The launch motor is used to launch the vehicle from the launch tube and acts as a boost phase. After 0.3 seconds of the launch, the launch tube is removed from the vehicle after which the flight motor is engaged which helps sustain the cruise speed (Mach 2) of the vehicle.

The specifications of the power unit are given in Table 2.

TABLE 2. MISSILE PROPELLANT DATA

Parameters	Mass/kg
Mass of Launch Motor + Propellant	0.87
Mass of Flight Motor + Propellant	6.65
Total Mass of Power Unit	7.52

The specifications of the boost propellant are given in Table 3.

TABLE 3. BOOST PHASE PROPELLANT DATA

Boost Propellant Specifications	
Weight of Propellant	1.75 kg
Burn time of Propellant	2.1 seconds
Liner Burn Rate of Propellant	7.8 mm/s

The specifications of the sustainer propellant are given in Table 4.

TABLE 4. SUSTAIN PHASE PROPELLANT DATA

Sustain Propellant Specifications	
Weight of Propellant	2.15 kg
Burn time of Propellant	4.6 seconds
Liner Burn Rate of Propellant	18 mm/s

VI. MASS PROPERTY VARIATION

The moments of inertia reflect how the mass of a rigid body is distributed. They manifest a body's resistance to being set into rotary motion or stopped once rotation is under way [2]. In case of rigid body a particular reference frame axis can be chosen where the product of inertia terms become zero and the inertia tensor can be written as (3), (4).

$$\bar{I} = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \tag{3}$$

$$\bar{I} = \begin{bmatrix} (y^2+z^2)m & 0 & 0 \\ 0 & (z^2+x^2)m & 0 \\ 0 & 0 & (x^2+y^2)m \end{bmatrix} \tag{4}$$

This reference frame axes is called the principal axes. In the case of our vehicle the principal axes frame coincides with the vehicle body frame axes therefore we will chose the body frame axes as our reference axes to simplify our inertia tensor

It should be noted that for the vehicle the mass of the propellant is constant expelled as it burns. As a result the moments of inertias and the center of gravity will be continuously changing with respect to time.

A. Finding Centre of Mass and Intertia Matrix

The missile consists of four sections as shown in Fig. 2 and Table 1. Referring to equation the inertia tensor is given by (4), it is assumed that the missile is cylindrical in shape and the masses of individual components act on the center of each section. The origins of x, y and z coordinates are on the center of the missile and the location of center of mass along x, y and z coordinates are on the respective axes.

Calculations below, along with Fig. 6, show the method adopted for calculating the center of gravity and the inertia matrix for a particular time segment (for the instant when the missile was fired). The same method was adopted to obtain a range of values of center of gravity and inertia matrix for different time segments. Finally a curve fitting tool in MATLAB was used to obtain the equations with respect to time for the variations in center of gravity and inertia matrix.

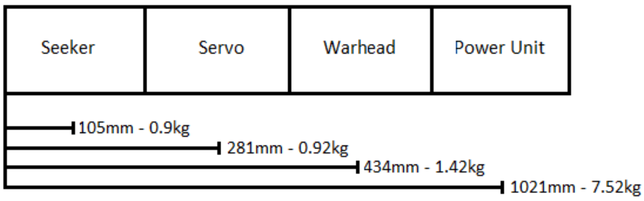

Fig. 6. Weights & dimensions of missile sections

$$W \times CG = \sum (W_{section} + D_{from\ reference}) \tag{5}$$

Where
W = total weight of missile
Wsection = Weight of individual section
CG = Centre of gravity
Dfrom reference = Distance from reference point

$$10.76 \times CG = (105 \times 0.9) + (281 \times 0.92) + (434 \times 1.42) + (1021 \times 7.52)$$

$$10.76 \times CG = 8647.22$$

$$CG = 803.64 \text{ mm}$$

$$I_{xx} = \sum (y^2 + z^2) \times m$$

$$I_{xx} = 0 \text{ kgm}^2$$

$$I_{yy} = \sum (x^2 + z^2) \times m$$

$$I_{yy} = 0.9 \times (0.69864)^2 + 0.92 \times (0.52264)^2 + 1.42 \times (36964)^2 + 7.52\,(21736)^2$$

$$I_{yy} = 1.239893504 \text{ kgm}^2$$

$$I_{zz} = \sum (x^2 + y^2) \times m$$

$$I_{zz} = 0.9 \times (0.69864)^2 + 0.92 \times (0.52264)^2 + 1.42 \times (36964)^2 + 7.52\,(21736)^2$$

$$Izz = 1.239893504 \text{ kgm}^2$$

Overall inertia matrix of vehicle for this particular case is given by (6)

$$\bar{I} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1.239893504 & 0 \\ 0 & 0 & 1.239893504 \end{bmatrix} kgm^2 \tag{6}$$

These calculations were repeated for eight time segments and the following results were obtained as shown by (7), (8), (9) and Fig. 7, 8, 9.

$$CG = 43.34 \times e^{-74.22\times t} + 760.4 \times e^{-0.03133\times t} \quad (7)$$

$$I_{yy} = = (-0.003 \times t^3) + (0.037 \times t^2) - (0.17 \times t) + 1.17 \quad (8)$$

$$I_{zz} = = (-0.003 \times t^3) + (0.037 \times t^2) - (0.17 \times t) + 1.17 \quad (9)$$

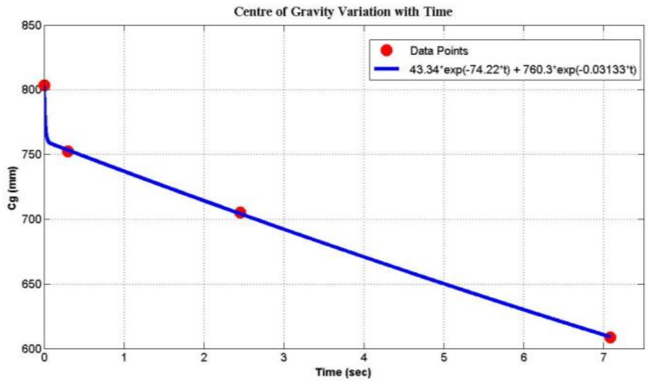

Fig. 7. Variation of missile's center of gravity with time

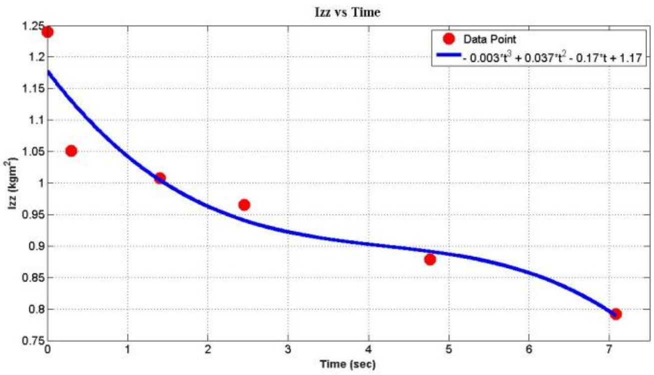

Fig. 8. Variation of I_{zz} with time

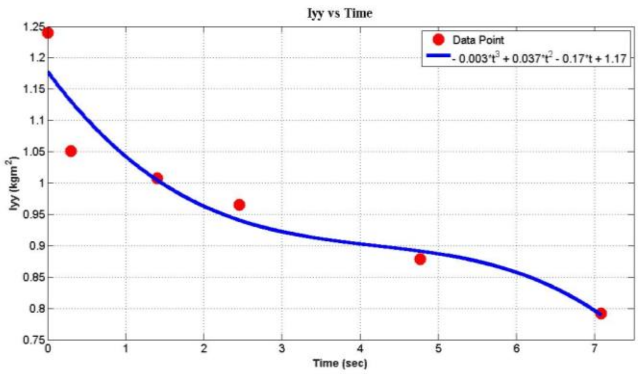

Fig. 9. Variation of I_{yy} with time

VII. GUIDANCE

A. Guidance Law

The PN guidance law issues acceleration commands, perpendicular to the instantaneous missile-target line-of-sight, which are proportional to the line-of-sight rate and closing velocity. [3]

Mathematically, the guidance law is given by (10).

$$n_c = N' V_c \dot{\lambda} \quad (10)$$

[3] uses a two dimensional, point mass missile-target engagement geometry to implement proportional navigation guidance as shown in Fig. 10.

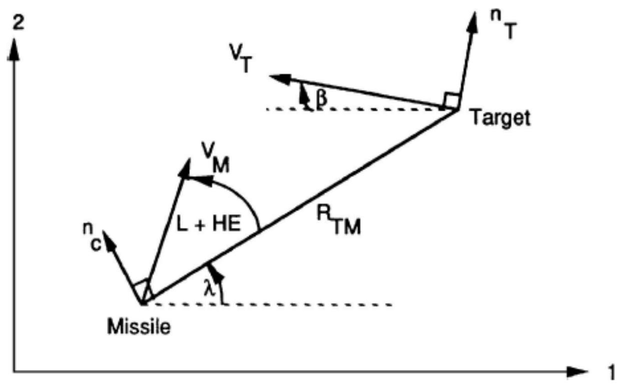

Fig. 10. Two dimensional missile-target engagement geometry

Using the geometry of the Fig. 10 and algebraic calculations [3] provides the following differential equations for a two dimensional missile-target engagement model.

Rate of change of line of sight is given as (11).

$$\dot{\lambda} = \frac{R_{TM1}V_{TM2} - R_{TM2}V_{TM1}}{R_{TM}^2} \quad (11)$$

While the closing velocity (negative of rate of change of distance between target and missile) is given by (12).

$$V_c = -\dot{R}_{TM} = \frac{-(R_{TM1}V_{TM1} - R_{TM2}V_{TM2})}{R_{TM}} \quad (12)$$

Equations (10), (11), (12) were used to define the guidance algorithm which was implemented in MATLAB. The algorithm guided the missile onto the collision triangle to hit the target.

B. Pitch Angle Required

The pitch commands to achieve required trajectories were obtain by applying the simple trigonometric formula (13) throughout the flight of the missile.

$$\tan\theta = \frac{RM2\ (next) - RM2\ (previous)}{RM1\ (next) - RM1\ (previous)} \quad (13)$$

VIII. STABILITY DERIVATIVES

Stability derivatives are constants calculated at some trim state for a flying vehicle. They predict the motion of the vehicle with first order equations near the trim state. These derivatives are used to calculate the forces and moments produced by the vehicle as required to calculate the A and B matrix of the state-space model.

DATCOM is a stability analysis software which was used to calculate the stability derivatives of the vehicle at different flight conditions. It uses Mach no, altitude, range of angle of attack, beta, mass and geometry parameters in order to find dynamic and static stability derivatives.

Once these derivatives are obtained they are converted into forces and moments to get A and B matrixes.

IX. CONTROLLER DESIGN

A. Control Law

LQR based controllers are full state feedback controllers which operate on static gains . They are relatively more robust than the usual PID Controllers.

For a system defined by the following matrix equation:

$$\dot{\boldsymbol{x}} = \boldsymbol{A}(t)\boldsymbol{x}(t) + \boldsymbol{B}(t)\boldsymbol{u}(t) \tag{14}$$

with the linear control law defined by:

$$\boldsymbol{u}(t) = -\boldsymbol{K}(t)\boldsymbol{x}(t) \tag{15}$$

The scalar objective function may be defined as:

$$J(t, t_f) = \int_t^{t_f} [\boldsymbol{x}^T(\tau)\boldsymbol{Q}(\tau)\boldsymbol{x}(\tau) + \boldsymbol{u}^T(\tau)\boldsymbol{R}(\tau)\boldsymbol{u}(\tau)]d\tau \tag{16}$$

Here x(τ) is the state matrix, u(τ) is the u matrix, Q(τ) is the state weighing matrix and R(τ) is the control cost matrix.

The goal of the LQR problem is to find a gain matrix K(t) such that the above mentioned objective function is minimized. The same theory may be used to design tracking systems. The theory for this involves the catenation of the state vector with the state error vector and applying the techniques for regulator design. The final result of this manipulation is that to design a tracking system using LQR technique, that reduces its error to zero in steady state, a feed forward gain matrix must be selected (in addition to the optimal feedback gain matrix designed for the regulator) such that [4]:

$$\boldsymbol{u}(t) = \boldsymbol{K}(t)\boldsymbol{e}(t) - \boldsymbol{K_d}(t)\boldsymbol{x_d^c} \tag{17}$$

The control law is then given as:

$$\boldsymbol{A}(t)\boldsymbol{x_d^c} = \boldsymbol{B}(t)\boldsymbol{K_d}(t)\boldsymbol{x_d^c} \tag{18}$$

.

B. Controller Design

The designing of the LQR then requires designing of the Q and R matrices. According to [5] the following criteria may be used.

$$\boldsymbol{Q} = \begin{bmatrix} \boldsymbol{q1} & & \\ & \ddots & \\ & & \boldsymbol{qn} \end{bmatrix} \qquad \boldsymbol{R} = \boldsymbol{\rho}\begin{bmatrix} \boldsymbol{r1} & & \\ & \ddots & \\ & & \boldsymbol{rn} \end{bmatrix} \tag{19}$$

$$\boldsymbol{qi} = \frac{\boldsymbol{1}}{\boldsymbol{tsi}(\boldsymbol{xi_{max}})^2} \qquad \boldsymbol{ri} = \frac{\boldsymbol{1}}{(\boldsymbol{ui_{max}})^2} \tag{20}$$

Nine separate LQR controllers for tracking pitch command were designed attributed by changing A and B matrix, feedback gains, feedforward gains and desired state (pitch) vectors for each second of the flight time. Each controller operates for a span of one second before switching to the other controller till the end of the flight.

The stability derivatives found via DATCOM were used to calculate A and B matrixes throughout the flight of the vehicle.

The required desired pitch state vector ($\boldsymbol{x_d^c}$) for a specific time interval were generated from the guidance command. MATLAB was used to solve the Algebraic Riccati equation to obtain the feedback gains and (21) was satisfied to obtain feedforward gains. These results formed the input for the state space model of the vehicle.

$$Ax_c^d = BK_{ff}x_c^d \tag{21}$$

Using the (19), (20) after multiple iterations, Q and R were designed as shown by (22)

$$\boldsymbol{Q} = \begin{bmatrix} \boldsymbol{1} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{1} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{260} \end{bmatrix} \qquad \boldsymbol{R} = [\boldsymbol{8.16}] \tag{22}$$

Fig. 11 on next page shows the Simulink block diagram of the entire model.

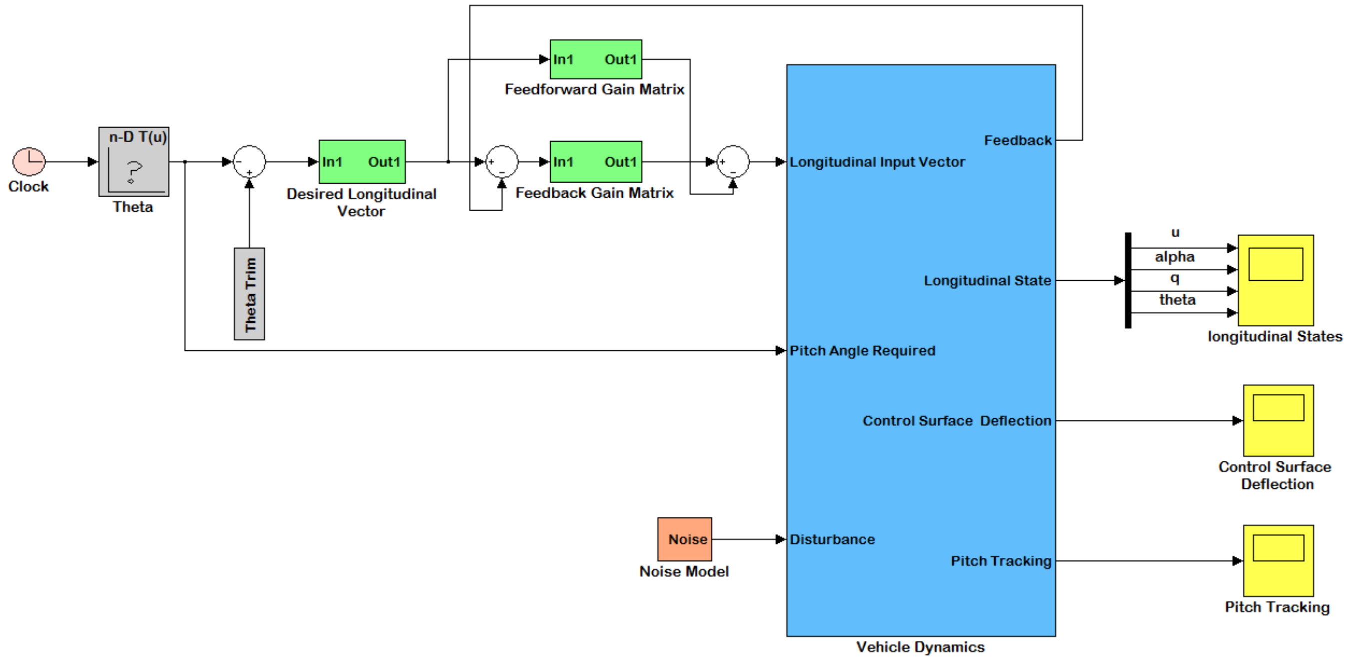

Fig. 11. Primary Simulink block diagram for simulations

X. Results

The program was run for a particular condition with the parameters given in Table 6.

TABLE 5. USER INPUT CONDITIONS

Missile Target Conditions	
Target Position	(3500,3200) m
Target Mach no.	2
Target Maneuver	5 g
Heading Error	23^0

The results of the simulation are shown in Fig. 12, 13, 14, 15, 16.

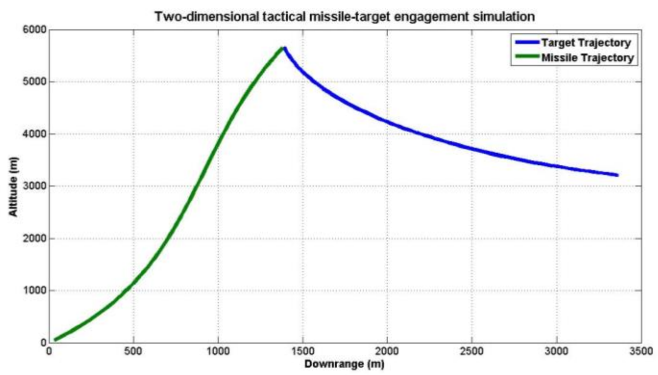

Fig. 12. Missile-target engagement trajectories

The blue curve in Fig. 12 shows that the target is performing a maneuver while the curve path of missile trajectory actually depicts that the missile is required to make corrections by applying acceleration commands in order to be on collision triangle and hit the target.

The acceleration commands required here shown in Fig. 13 are much high then the ideal case because the missile needs to be on the collision triangle and to do so it required acceleration. It should be noted that the acceleration commands are monotonically decreasing which is a great benefit for the system because in the end the fuel in the system is much depleted and it would be more preferable to apply low acceleration commands.

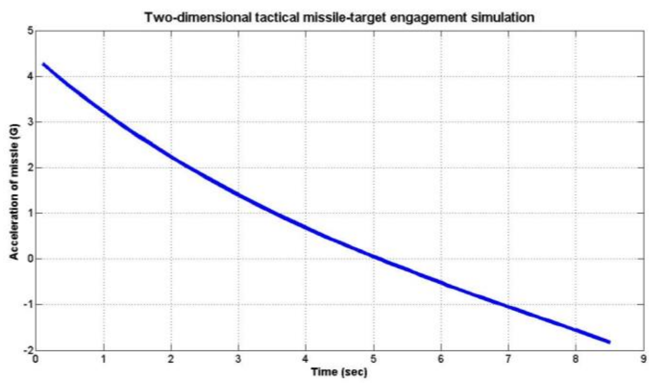

Fig. 13. Acceleration commands

The deviation from the longitudinal states along the flight are depicted in Fig. 14.

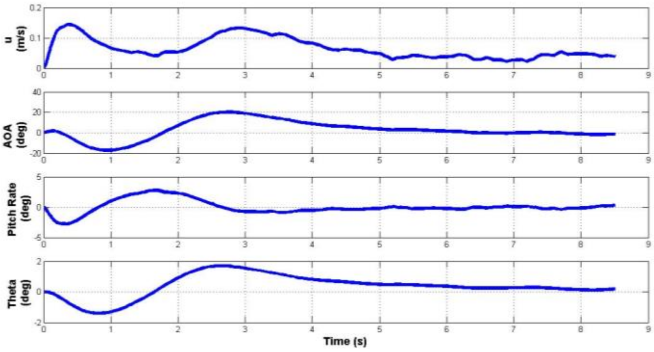

Fig. 14. Variations from longitudinal states

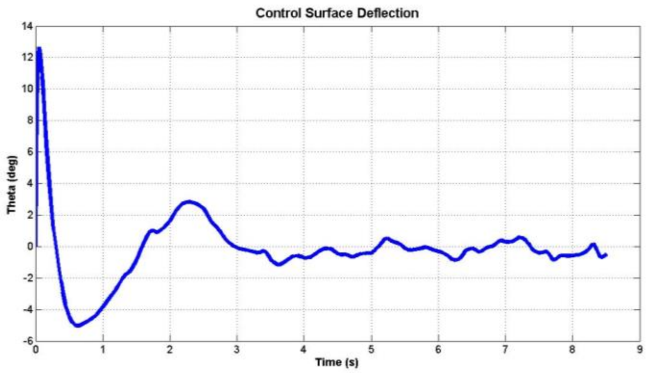

Fig. 15. Control surface deflection to track pitch

The control surface deflections required to track the pitch are given above in Fig. 15, while the tracking of pitch is depicted below in Fig. 16.

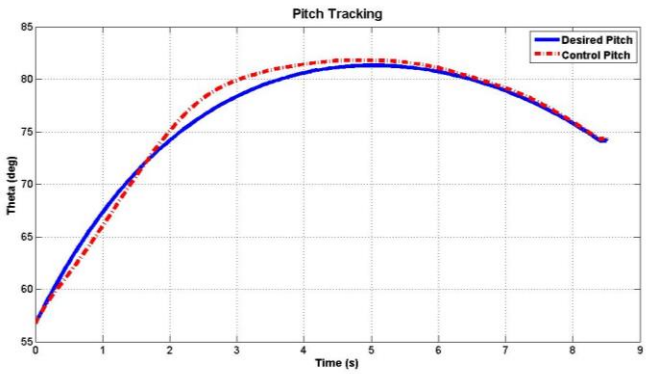

Fig. 16. Tracking of pitch commands

XI. CONCLUSION

A Pitch Tracking Controller for a MANPADS missile has been designed to intercept a target as per the commands provided by the guidance algorithm. The LQR based controller was used to track the required pitch commands provided. The designed controller provides much satisfactory response to tracking commands and minimizing disturbance torques within the defined limits.

XII. FUTURE WORK

Future work may be carried out in research towards nonlinear dynamic models and nonlinear control. Also other techniques for control could be applied such as H-infinity loop shaping. A recommended line of work is the selection of a roll stabilized vehicle. Applying this system in programs like X-PLANE can be a good addition for its future.

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