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Mathematical Analysis of Vehicle Longitudinal Dynamics

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***Abstract*—This research paper presents the longitudinal dynamic model of a vehicle. The two major components of the longitudinal vehicle model are the powertrain dynamics and the vehicle longitudinal dynamics. The automotive powertrain is divided into the following sub-models: an engine model, a torque converter model, an automatic transmission model, and a drivetrain model. The vehicle longitudinal dynamics are mainly influenced by the longitudinal tire forces, aerodynamic drag forces, rolling resistance forces and the gravitational forces.**

***Keywords*—Vehicle longitudinal dynamics; engine model; powertrain model; drivetrain model.**

I. Introduction

There has been a great deal of development in the area of automotive powertrain system dynamics for control during automotive transmission up-shifting and down-shifting. A complex automotive powertrain model based on the combination of Cho and Hedrick [1] and Rajamani [2] models is redeveloped based on the physical principles and captures the powertrain dynamics in the continuous-time domain which consists of four states. Several physical assumptions are made to simplify the mathematical modeling. To further simplify the model the gear up-shifting and down-shifting in the transmission model is controlled by transmission look-up tables [2, 3] rather than a complex planetary gear system model. The look-up tables contain the gear up-shifting and down-shifting schedules and the gear transmission takes place using the information from these look-up tables. This approach has significantly saved the modelling and computation time. The simulation results, using these look-up tables, will be compared with the other automatic transmission models for the validation purpose.

Steady-state engine maps are constructed in this study. Usually the data for the engine maps is provided by the manufacturers, but in this study the steady-state engine maps have been constructed off-line by computation using the MATLAB software.

II. Vehicle Model

Fig. 1. shows a vehicle model which comprises of the sub-models developed in this study. The block diagram shows the inputs and outputs to each sub-model. The power generated in the engine by combustion of fuel travels from engine to the wheel through the crankshaft, torque convertor, transmission system, and driveline. Finally, the vehicle longitudinal motion is captured in continuous time domain using a set of differential equations.

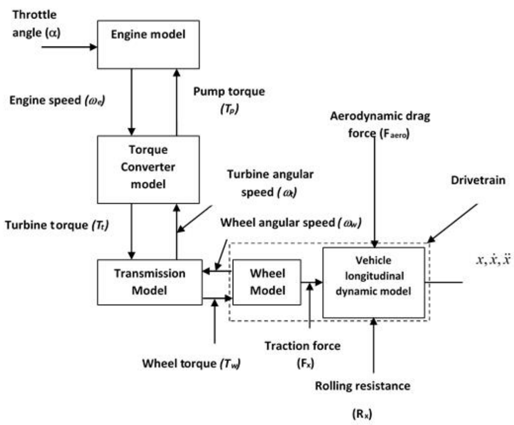

Fig. 1. Block diagram for a vehicle

The subsequent sections present the modeling of an engine model, an automatic transmission model, and a drivetrain model.

A. Engine Dynamics

Most of the engine models are concerned with combustion dynamics, pollution formation, or engine dynamics control. The engine model investigated in this study is a dynamic engine control model. The type of engine model developed is called a mean value model [1, 2]. A mean value model is a mathematical engine model which is intermediate between large cyclic simulation models and simplistic transfer function models. It predicts the mean values of the engine states e.g. engine speed and the intake manifold pressure. The engine model is developed as torque-producing device with one inertia where the engine torque is the function of the throttle angle (α), mass flow rate of fuel ($\dot{m}_f$), and the engine speed (ω_e).

In this study a 3.8L spark-ignition engine model with six cylinders and a five-speed automatic transmission for a typical front-wheel-drive passenger car is developed where the input to the engine model is the throttle angle (α). The proposed mathematical engine model is used to control the longitudinal dynamics of the vehicle and can be divided into different subsystems namely throttle body, intake manifold, fuel injection, combustion chamber and torque production, and crank shaft rotation. The input to the throttle body is the throttle angle which can be controlled instantly and contains no dynamic elements. The output from the engine dynamic is engine speed which is an input to the torque converter. The engine speed is fed back to the combustion model in order to compute the volumetric efficiency.

A continuous time domain, 2 state engine model based on the combination of Cho and Hedrick [1] and Rajamani [2] models is developed, where the two states are intake manifold pressure (p_{man}) and the engine angular speed (ω_e). The intake manifold can be defined as the volume between the throttle plate and the intake valves of the cylinder. The mass of the intake manifold (m_{man}) can be related to the intake manifold pressure (p_{man}) using the ideal gas equation.

$$p_{man}V_{man} = m_{man}RT_{man} \tag{1}$$

where, T_{man} is the manifold temperature which assumed as constant, R is the gas constant of air, and V_{man} is the intake manifold volume.

Taking derivative of (1) gives the state equation for the intake manifold pressure.

$$\dot{p}_{man} = \frac{RT_{man}}{V_{man}}(\dot{m}_{man}) \tag{2}$$

where, $\dot{m}_{man}$ is the mass flow rate in the intake manifold. When the law of conservation of mass is applied to the intake manifold then we get

$$\dot{m}_{man} = \dot{m}_{ai} - \dot{m}_{ao} \tag{3}$$

where, $\dot{m}_{ai}$ and $\dot{m}_{ao}$ represent mass flow rate in and out of the intake manifold i.e. through the valve and into the cylinder respectively. Then (2) becomes

$$\dot{p}_{man} = \frac{RT_{man}}{V_{man}}(\dot{m}_{ai} - \dot{m}_{ao}) \tag{4}$$

The throttle body describes the transformation of throttle angle inputs to mass rates of air entering the intake manifold $\dot{m}_{ai}$, and is expressed by

$$\dot{m}_{ai} = MAX \cdot TC(\alpha) \cdot PRI \tag{5}$$

where, MAX is the maximum flow rate of air through the throttle body. TC is the normalized throttle characteristics and is a function of throttle angle α (Cho and Hedrick, 1989).

$$TC = \begin{cases} 1-\cos(1.14459\times\alpha - 1.0600) & for\alpha \le 79.46^0 \\ 1 & for\alpha > 79.46^0 \end{cases} \tag{6}$$

PRI is the normalized pressure ratio influence and is a function of manifold to atmospheric pressure ratio. PRI can be defined as [1].

$$PRI = 1 - \exp\left[9\left(\frac{p_{man}}{P_{atm}} - 1\right)\right] \tag{7}$$

$\dot{m}_{ao}$, the mass rate of air leaving the manifold (and thus, entering the combustion chamber), is dependent upon engine characteristics, e.g. displacement volume V_d, volumetric efficiency η_{vol}, intake manifold pressure p_{man}, and the engine speed ω_e and is described by [2].

$$\dot{m}_{ao} = \eta_{vol}\frac{\omega_e}{4\pi}V_d\frac{p_{man}}{RT_{man}} \tag{8}$$

Volumetric efficiency η_{vol} is a complex function of the engine state ω_e.

The second state concerns with the rotational dynamics of an engine. The crank shaft follows the conservation of moment about a rigid shaft.

$$I_e\dot{\omega}_e = T_i - T_f - T_a - T_p \tag{9}$$

where, T_i is the engine combustion torque which is dependent on the ignition of a cylinder charge of air, fuel and residual gas [4], T_f is the engine friction torque, T_a is the accessory torque and T_p is the pump torque and represents the external load on the engine. I_e is the effective inertia of engine. Equation (9) can be written as [2].

$$I_e\dot{\omega}_e = T_{net} - T_{load} \tag{10}$$

where, $T_{net} = T_i - T_f - T_a$, is the net engine torque after losses and depends on the dynamics in the intake and exhaust manifolds of the engine and on the accelerator input from the driver. T_{load} is the load on the engine which represents the pump torque T_p. T_{load} is provided by the torque converter which couples the engine to the transmission.

The indicated torque T_i is generated by combustion and can be represented by [2].

$$T_i = \frac{H_u\eta_i\dot{m}_f}{\omega_e} \tag{11}$$

where, H_u is the fuel energy constant, η_i is the thermal efficiency multiplier, and accounts for the cooling and the exhaust system losses, and $\dot{m}_f$ represents the fuel mass flow rate into the cylinder.

$$\dot{m}_f = \frac{\dot{m}_{ao}}{\lambda \cdot L_{th}} \tag{12}$$

where, L_{th} is the stoichiometric air/fuel mass ratio for gasoline (fuel) and λ is the air/fuel equivalence ratio. The torque due to friction (T_f), is curve-fitted from experimental data and is the function of engine speed ω_e [1].

$$T_f = 0.1056\omega_e + 15.10 \tag{13}$$

B. Torque Converter

The output from the engine dynamic model is engine speed which is in turn an input to the torque converter. A torque converter consists of a pump (driving member), a turbine (driven or output member), and a stator (reaction member). The pump is attached to the engine and turns at engine speed, and the turbine is the input to the transmission [1]. The torque converter uses the engine speed and transmission speed to compute the pump and turbine torques. The turbine torque is used to accelerate the mechanical elements of the transmission.

The equations for the pump torque T_p and turbine torque T_t can be found in [1].

For converter mode ($\omega_t/\omega_p < 0.9$), the pump and turbine torques are given by:

$$T_p = 3.4325es - 3\omega_p^{\,2} + 2.2210x10^{-3}\,\omega_p\omega_t - 4.60141x10^{-3}\,\omega_t^{\,2} \tag{14}$$

$$T_t = 5.7656x10^{-3}\,\omega_p^{\,2} + 0.3107x10^{-3}\,\omega_p\omega_t - 5.4323x10^{-3}\,\omega_t^{\,2} \tag{15}$$

For fluid coupling mode ($\omega_t/\omega_p \geq 0.9$), the pump and turbine torques are given by:

$$T_p = T_t = -6.7644x10^{-3}\,\omega_p^{\,2} + 32.0024x10^{-3}\,\omega_p\omega_t - 25.2441x10^{-3}\,\omega_t^{\,2} \tag{16}$$

The output from the torque converter i.e. turbine torque is the input to the transmission model.

C. Transmission Model

The output from the torque converter model is the turbine torque which is in turn the input to the transmission model. The transmission output torque is used to accelerate the vehicle against the aerodynamic drag load, the road friction, and road gradient loads. The transmission model considered is a five-speed automatic transmission model and also includes the final drive reduction in the differential; therefore, the operating gear ratio also includes the final gear reduction in the differential. In this study instead of the complex mathematical automatic transmission model look-tables (explained below) have been used for gear up shift and down shift operation.

If the torque transmitted to the wheels is T_w then for the steady-state operation under the first, second or higher gears of the transmission, the torque transmitted to the wheel is [2].

$$\tau \dot{T}_{wheel} + T_{wheel} = \frac{1}{R_g R_d} T_t \tag{17}$$

Similarly, the relation between the transmission and wheel speeds is

$$\tau \dot{\omega}_t + \omega_t = \frac{\omega_w}{R_g R_d} \tag{18}$$

where, R_g is the gear ratio on the transmission (g = 1, 2, 3, 4, 5) and R_d is the final gear reduction in the differential. The value of R_g depends on the operating gear and increases as the gear shifts upwards.

In order to avoid the complexities and to save the modelling and computation time mathematical model of a transmission system has been replaced by look-up tables. The look-up tables [2, 3] has been used for a 5-speed automatic transmission up-shifting and down-shifting as shown in Fig. 2. The operating gear is obtained by a gear shift schedule in [3] look-up table in Fig. 2 which is the function of the throttle angle and the longitudinal vehicle speed. The control strategy to decide and select the up-shifting or down-shifting control is the function of the acceleration of the vehicle [5].

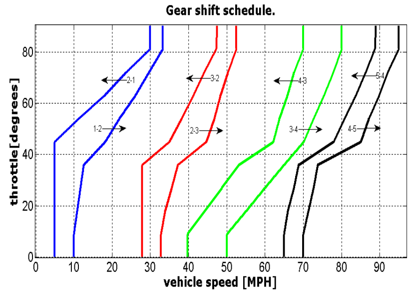

Fig. 2. Gear up & down shifting schedules for an automatic transmission, [3]

D. Drivetrain Model

A drive-train serves as a simple power transfer device between the transmission system and the vehicle. A two state drive train model has been presented [2] where the two states are front wheel angular speed (ω_{wf}) and the longitudinal vehicle speed ($\dot{x}$) of the vehicle.

1) Wheel Dynamics: The output from the transmission model, as described in (17), is the wheel torque which is in turn the input to the wheel model. When the vehicle is moving, the wheel is subjected to the ground forces as shown in Fig. 3. The ground forces on a wheel are normal force F_z which is due to the weight and load of the vehicle, the longitudinal traction/braking force F_x which is generated during driving and braking, and the lateral force F_y which helps the vehicle in turning and maneuvering, or in the case of minimizing the disturbing effect of wind or other side forces. The lateral force is not the part of this study, therefore, only the longitudinal force F_x is considered in the modelling.

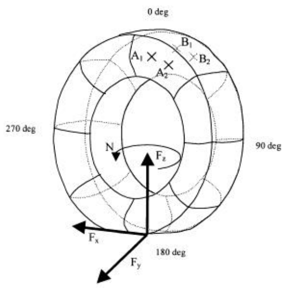

Fig. 3. Free body diagram of a wheel [6]

During acceleration or braking maneuver, the contact patch (explained in next section) between tire and ground begins to slip. The longitudinal slip can be defined as the difference between the actual longitudinal velocity at the axle of the wheel V_x and the equivalent rotational velocity $r_{eff}\,\omega_w$ of the tire. The longitudinal slip ratio is defined as [7].

$$\sigma_x = \frac{\mathrm{r}\omega_w - V_x}{V_x} \text{ (during braking)} \qquad (19)$$

$$\sigma_x = \frac{\mathrm{r}\omega_w - V_x}{\mathrm{r}\omega_w} \text{ (during acceleration)} \qquad (20)$$

where, V_x is the longitudinal speed of the vehicle, r is the rolling radius of the free-rolling tire, and ω_w is the angular speed of the wheel.

A front wheel drive vehicle is considered in this study and the differential equation relating the wheel angular speed ω_{wf}, wheel torque T_w, and the longitudinal tire force F_{xf} is

$$I_w \dot{\omega}_{wf} = T_w - r_{eff} F_{xf} - R_{xf} - T_{bf} \qquad (21)$$

where, r_{eff} is the effective radius of the wheel which is the ratio of the longitudinal speed of the tire center to the angular speed of the tire, R_{xf} is the rolling resistance in the front wheel and T_{bf} is the front brake torque.

When the wheel torque is applied, a friction torque due to the friction force between road and tire and rolling resistance is generated. The wheel rotational dynamic is controlled by this torques as defined by (21).

A simple model for rolling resistance has been developed in [1] powertrain model and has been chosen for this study.

$$R_{xf} = R_{xr} = 0.001546 \times m \times g[N.m] \qquad (22)$$

2) Longitudinal Vehicle Dynamcis: The external longitudinal forces acting on the vehicle include aerodynamic drag forces, gravitational forces, longitudinal tire forces and rolling resistances. A force balance along the vehicle longitudinal axis yields [1].

$$m\ddot{x} = F_{xf} - F_{aero} - \frac{R_{xf}}{r_{eff}} - mg\sin\theta \qquad (23)$$

where, m = mass of the vehicle, x = vehicle displacement [m], F_{xf} = longitudinal tire force at the front tire, F_{aero} = longitudinal aerodynamic drag force, g = gravitational acceleration, θ = gradient of the road. The aerodynamic drag force on a vehicle can be represented as [2]

$$F_{aero} = \frac{1}{2}\rho C_d A_F (V_x + V_{wind})^2 \qquad (24)$$

where, ρ = air density , C_d = aerodynamic drag coefficient, A_F = frontal area, V_x= vehicle longitudinal velocity, and V_{wind} = wind velocity.

The term A_F can be defined as the area calculated from the vehicle width and height for passenger cars [8].

$$A_F = 1.6 + 0.00056(m - 765) \qquad (25)$$

where, the vehicle mass (m) is in range of 800-2000 k. This longitudinal tire force on the wheels is the force which moves the vehicle forward. Experimental results have shown that the longitudinal tire forces depends on [9].

a) the slip ratio (σ_x)
b) the normal load on the tire
c) the friction coefficient of the tire-road interface

Longitudinal traction forces at the front and rear tire as the function of slip ratio are:

$$F_{xf} = C_{\sigma f}\,\sigma_{xf} \qquad (26)$$

$$F_{xr} = C_{\sigma r}\,\sigma_{xr} \qquad (27)$$

where,

$C_{\sigma f}$ = longitudinal tire stiffness of the front tire = 80000 N,

$C_{\sigma r}$ = longitudinal tire stiffness of the rear tire = 80000 N

For the purpose of detailed analysis, the longitudinal vehicle model will be subjected to gear up-shifting and down-shifting using the throttle control and on a gradient of the road. Simulations will be run to analyze the transient and steady-state behavior of the powertrain dynamic model based on the above control input signals.

III. Result and Discussion

A gear shift schedule [3] shown in Fig. 2 has been adopted for the transmission up-shifting and transmission down-shifting where the gear schedule is the function of throttle angle and the vehicle longitudinal speed. Furthermore the simulations have been carried out using the [3] look-up table. Fig. 4 shows engine speed for 5-speed automatic transmission at full throttle using the [3] gear shift schedule as shown in Fig. 2. The gear ratios used in Fig. 4 are the same as in the [3] model as the gear shift schedule is from the same model.

Fig. 5 shows the corresponding vehicle longitudinal speeds for the engine speed shown in Fig. 4. The simulation result in Fig. 5 has been validated with [3] model. The gear shift schedule is controlled by the throttle angle and vehicle longitudinal speed as shown in Fig. 2. The throttle input to the engine model is 100 % and the gear up-shifts take place at the velocities shown in Fig. 2.

Since the proposed powertrain model has been reproduced and validated against the experimental results [1, 3] and the look-up table approach for the automatic transmission has also been justified, therefore, this model is useful in the development of a longitudinal dynamic control system for the vehicles equipped with ACC system.

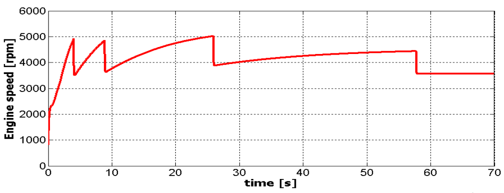

Fig. 4. Engine speed during 1-2-3-4-5 gear up-shift

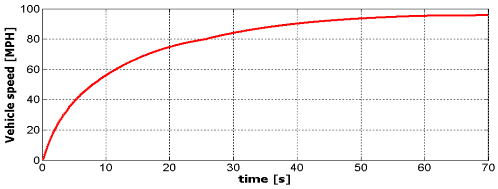

Fig. 5. Vehicle Speed during 1-2-3-4-5 gear up-shift

TABLE I. POWERTRAIN PARAMETERS

Powertrain Parameters		
Engine displacement	V_d	0.0038 m^3
Intake manifold volume	V_{man}	0.0027 m^3
Universal gas constant	R	287
Manifold temperature	T_{man}	293 K
Max. Flow rate of air in the intake manifold	MAX	0.1843 kg/s
Air/fuel equivalence ratio	Λ	1
Stoichiometric air/fuel mass ratio for gasoline(fuel)	L_{th}	14.67
Fuel energy constant	H_u	4.3e7 J/kg
Thermal efficiency	η_i	0.32
Moment of inertia of engine	I_e	0.1454 $kg.m^2$
1^{st} gear speed reduction ratio [3]	R_1	0.3184
2^{nd} gear speed reduction ratio [3]	R_2	0.505
3^{rd} gear speed reduction ratio [3]	R_3	0.73
4^{th} gear speed reduction ratio [3]	R_4	1
5^{th} gear speed reduction ratio [3]	R_5	1.3157
Final drive speed reduction ratio [3]	R_d	0.3257
Moment of inertia of wheel	I_w	2.8 $kg.m^2$
Effective radius	r_{eff}	0.3 m
Longitudinal tire stiffness (for both tires)	$C_{\sigma f}$	80000 N
Air density	ρ	1.225 kg/m^3
Aerodynamic drag coefficient	C_d	0.4
Wind velocity	V_{wind}	10 m/s
Mass of the vehicle	M	1644 kg

Table I shows all the powertrain parameters used in the entire vehicle model. Most of the parameters are taken from [1] model.

IV. CONCLUSION

In this study a 5-speed automatic transmission powertrain model has been developed for the longitudinal dynamic control of a vehicle. The proposed model captures the powertrain dynamics in the continuous-time domain. A simplified approach for gear up-shifting and down-shifting using the gear schedule maps (Fig. 2) has been proposed where the transmission gear up-shifting and down-shifting is triggered using the look-up maps based on the throttle input and vehicle's longitudinal speed information [3]. The simulation results obtained from the proposed model have been validated against previous studies [1]. It has been observed that the vehicle longitudinal dynamics is precisely controlled using these input commands.

V. ACKNOWLEDGMENT

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