Formation Flight Control of Aircraft with Relaxed Longitudinal Stability

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Abstract* — This paper focuses on developing a controller for autonomous formation flying of aircraft with relaxed static stability, to minimize drag. Wake was modelled by applying Burnham-Hallock vortex model on the single horseshoe vortex and distribution was calculated using strip theorem which was further used to calculate an optimum separation of leader and follower aircraft, at which drag is minimum. An LQI based control and stability augmentation system was developed for the reference aircraft to cater for relaxed static stability as well as to track velocity and altitude reference. Formation hold controller was subsequently developed to maintain the desired optimum separation between leader and follower aircraft. Both PID and LQ control schemes were developed for this purpose. Control systems were verified in a MATLAB® - Simulink® based simulation.

***Keywords*—Formation Flying Control, Drag Reduction, Fuel Saving, LQ control, Relaxed Longitudinal Stability**

I. NOMENCLATURE

$\hbar_E$	=	Engine's angular momentum
R_I^B	=	Rotation matrix, inertial to body frame
$C_{x_t}, C_{y_t}, C_{z_t}$	=	Coefficient of force in x,y,z-direction
$C_{L_t}, C_{M_t}, C_{N_t}$	=	Coefficient of roll, pitch, yaw moments
S	=	Wing planform area
Q	=	Dynamic pressure
b	=	Wingspan
c	=	Chord
$\rightarrow$	=	Angle of attack
$\downarrow$	=	Sideslip angle
D_N	=	North displacement
D_E	=	East displacement
Pow	=	Power (%)
Γ	=	Circulation
r_c	=	Vortex core radius

II. INTRODUCTION

Formation flying is a flight maneuver in which two or more aircraft fly in a specific relative orientation at a certain relative distance to each other. This maneuver not only provides drag reduction, but is also used in other applications, such as air-to-air refueling, defensive strategies etc. But the aerodynamic industry is far from attaining the full benefits of formation flying because the upwash from the leader aircraft causes highly non-linear and very large moments on the follower aircraft, hence it is difficult to perform this maneuver safely and efficiently.

This paper focuses on developing a controller for autonomous formation flying of aircraft. To make sure maximum drag reduction, wake of the leader aircraft is modelled and optimum separation is calculated for the follower aircraft. The developed controllers are tested and verified through simulation.

The research will limit itself to a 'two-aircraft' formation, while applying the 'Leader-Follower' approach to develop the formation control.

III. LITERATURE REVIEW

There are various approaches to formation flying control that have been adopted in previous researches. For the purpose of this paper, leader-follower approach [1] has been selected. This approach was selected after considering a few major advantages it offers over the other approaches. Unlike virtual leader approach [2], leader-follower approach has constant communication with the leader aircraft which will provide a more practical control system. It also has an advantage over the virtual reference point approach [3] such that the leader-follower approach enables tracking of orientation of the leader aircraft, therefore, resulting in a better control system of relative orientation of the follower aircraft. The key drawback of this approach is error propagation. Since this research will focus on only two aircraft formation, hence error propagation can be overlooked in comparison to the advantages it presents.

In previous researches pertaining to this field [4-6] classical control schemes were used for formation flying of C-130 aircraft. [7] developed extremum seeking formation control for C-5 aircraft, while many researches like [8] pertain to formation of UAVs (Unmanned Aerial Vehicles).

IV. METHODOLOGY

The methodology adopted for this research entails developing a *mass-varying* dynamical model of the reference aircraft (Appendix A). The developed model is then trimmed at a preset velocity and altitude and linearized at this trim point. Wake of the leader aircraft is modelled to calculate the forces and moments distribution on the follower aircraft. This enables calculation of optimum separation at which drag is minimum. Control and stability augmentation system is developed to improve longitudinal stability and maintain trim velocity and altitude. Subsequently, formation hold control system is developed to maintain optimum separation. Next the formation hold control is developed. Control schemes are verified using a MATLAB® - Simulink® based simulation

V. DYNAMICAL MODELLING

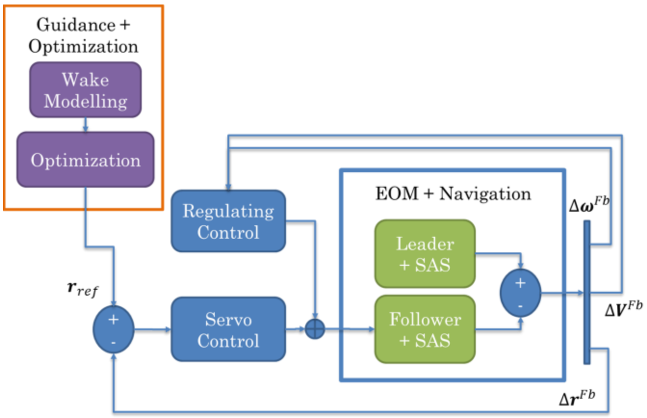

Fig. 1. Methodology Layout

This section focusses on the development of mass-varying dynamical model of the reference aircraft, F-16. The complete non-linear model will be an amalgamation of kinetic, kinematic, gravity, atmospheric, engine and aerodynamic models.

A. Assumptions

Following assumptions are made while deriving the dynamical model of F-16.

1) Aircraft is a rigid body.
2) Mass and inertia tensor vary with time (details in appendix).
3) The earth is considered as flat and non-rotating, inertial frame of reference.
4) Wind gust effects are not taken into account, i.e. the air is assumed to be at rest w.r.t. the surface of the earth.
5) The Mass distribution is symmetric about XZ plane of Body Frame.

B. Kinetics

Firstly, force and moment equations are developed for variable mass and inertia tensor.

Forces, F, and moments, M, are modelled as:

$$\mathbf{F} = \frac{d}{dt}(m\mathbf{V})\Big|_I = \frac{d}{dt}(m\mathbf{V})\Big|_B + \boldsymbol{\omega}\times(m\mathbf{V})$$

$$\mathbf{M} = \frac{d}{dt}(\mathbf{H})\Big|_I = \frac{d}{dt}(\mathbf{H})\Big|_B + \boldsymbol{\omega}\times(\mathbf{H}) \qquad (1)$$

C. Gravity and Atmosphere Model

WGS – 84 model is used for gravitational acceleration. The gravitational force acts along z-axis in inertial frame, hence the weight, F_w, is modelled as:

$$\mathbf{F_W} = R_I^B\,\mathbf{F_W}|_I = R_I^B\begin{bmatrix}0\\0\\mg\end{bmatrix} = \begin{bmatrix}-mg\sin\theta\\ mg\cos\theta\sin\phi\\ mg\cos\theta\cos\phi\end{bmatrix}$$

ICAO Standard Atmosphere model is used for atmospheric modelling.

D. Engine Model

The detailed engine model is provided in [9-11].

By definition of TSFC (Thrust Specific Fuel Consumption), c_t, we have

$$\dot{m} = -\dot{m}_f = -\frac{c_t F_T}{g} \qquad (2)$$

Thrust vector, F_T, is assumed parallel to the body x-axis, hence:

$$\mathbf{F_T} = \begin{bmatrix} F_T & 0 & 0 \end{bmatrix}^T$$

Since engine is mounted under fuselage hence, the moment due to thrust, M_T, is calculated as follows:

$$\mathbf{M_T} = \overbrace{\begin{bmatrix} F_T & 0 & 0 \end{bmatrix}^T \times \begin{bmatrix} x_T & 0 & z_T \end{bmatrix}^T}^{\text{due to thrust of engine}} + \overbrace{\begin{bmatrix} h_E & 0 & 0 \end{bmatrix}^T \times \boldsymbol{\omega}}^{\text{due to angular momentum of engine}}$$

E. Aerodynamics Model

The forces and moments due to aerodynamic forces are represented as;

$$X = QSC_{X_T} \qquad Y = QSC_{Y_T} \qquad Z = QSC_{Z_T}$$
$$L = QSbC_{L_T} \qquad M = QS\bar{c}C_{M_T} \qquad N = QSbC_{N_T}$$

The aerodynamic coefficients of the reference aircraft (F-16) have been derived from wind-tunnel tests conducted at the NASA Ames and Langley Research Centers [9]. This data is valid for the following range:

$-10 \le \alpha \le 45$ degrees and $-30 \le \beta \le 30$ degrees.

F. Complete Non-Linear Model

State vector

$$x = \begin{bmatrix} V_T & \alpha & \beta & \phi & \theta & \psi & p & q & r & D_N & D_E & h & pow & m \end{bmatrix}^T$$

Control Inputs

$$u = \begin{bmatrix} \delta_T & \delta_E & \delta_A & \delta_R \end{bmatrix}^T$$

Model

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u})$$

G. Linearization

Trimming and linearization of F-16 is presented in [12, 13] but for a constant mass. This work is modified using the engine model, and terms pertaining to mass variation were added in the aerodynamics model.

The aircraft is trimmed about steady-level flight conditions.

$$\begin{array}{lll} m_o \neq 0 & h_o \neq 0 & \\ u_o \neq 0 & v_o = 0 & w_o \neq 0 \\ p_o = 0 & q_o = 0 & r_o = 0 \\ \phi_o = 0 & \theta_o \neq 0 & \psi_o = 0 \end{array}$$

The non-linear model is then linearized by using gradient based scheme;

$$\dot{x} = \left(\nabla_x f(x,u)|_{x=x_{trim};u=u_{trim}}\right)x + \left(\nabla_u f(x,u)|_{x=x_{trim};u=u_{trim}}\right)u$$

VI. Wake Modelling

The wake of leader aircraft is primarily shaped by wing tip vortices. These wing tip vortices have three components; upwash, sidewash and rolling moment. As time progresses, vortices increase in size and decay in terms of energy. This section focuses on modelling these vortices in accord to their upwash, sidewash and rolling moment distributions. This model will be used to calculate forces and moments on the follower aircraft due to wake and will facilitate in determination of the optimum separation for minimum drag.

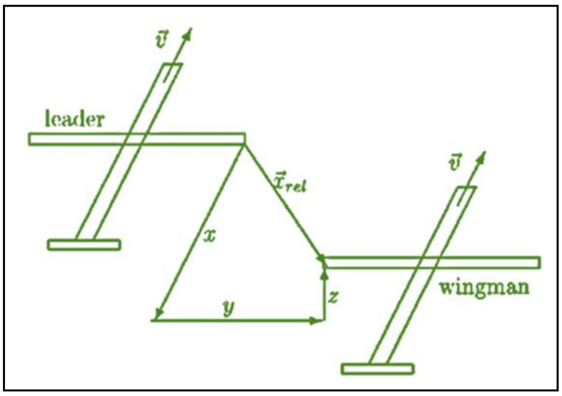

Fig. 2. Notation Description [7]

A. Vortex Model

The purpose of vortex model is calculate vortex circulation. Many vortex models have been developed and presented in the past literature. This research incorporates single horseshoe model [7] with Burnham-Hallock vortex model [14-18]. Using single horseshoe vortex emphasizes that vortex decay was not considerable, since the leader and follower aircraft have relatively small tip-to-tip separation. The Burnham-Hallock model was used since it was developed as a simplified empirical fit based on experimental data, and is therefore, more accurate than other theoretical derivation based models.

The circulation was modelled as:

$$\Gamma = \Gamma_v \left(\frac{r^2}{r^2 + r_c^2} \right)$$

B. Wake Distribution

The upwash, W_{wake}, and sidewash, V_{wake}, distributions (for right and left wing vortices), respectively are as follows [7]:

$$W_{wake} = \frac{\Gamma}{4\pi} \left\{ \begin{array}{c} \left[\frac{y}{y^2+z^2+r_c^2} \left(1 + \frac{x}{\sqrt{x^2+y^2+z^2}} \right) \right] \\ + \left[\frac{y+b}{(y+b)^2+z^2+r_c^2} \left(1 + \frac{x}{\sqrt{x^2+(y+b)^2+z^2}} \right) \right] \end{array} \right\}$$

$$V_{wake} = \frac{\Gamma}{4\pi} \left\{ \begin{array}{c} \left[\frac{z}{y^2+z^2+r_c^2} \left(1 + \frac{x}{\sqrt{x^2+y^2+z^2}} \right) \right] \\ + \left[\frac{z}{(y+b)^2+z^2+r_c^2} \left(1 + \frac{x}{\sqrt{x^2+(y+b)^2+z^2}} \right) \right] \end{array} \right\}$$

C. Forces and Moments on Follower

Average rolling moment, L_{wake}, over the whole span is determined by applying modified strip theory [7]

$$L_{wake} = -\frac{m}{2} \rho_0 V_\infty a_0 \int_0^b W_{wake}(x, y+s, z) c(s) Q(s) s \, ds \tag{3}$$

Where;

$$Q(s) = \frac{\pi}{4} \sqrt{1 - \left(\frac{2(s-b/2)}{b} \right)^2}, \text{ and}$$

$$m = \frac{1}{1+(2a_0/\pi AR)(1+\varepsilon)}$$

The average upwash force and moment can be determined by integrating the wake distribution over the whole span

$$\bar{W}_{wake} = \frac{1}{b} \int_0^b W_{wake}(x, y+s, z) c(s) \, ds \tag{4}$$

D. Effect of Wake on Follower Dynamics

Follower aircraft dynamics have an added component due to wake [7],

$$\dot{x} = Ax + Bu + Fu_{wake} \tag{5}$$

Where the F matrix dictates the affect of wake on dynamics,

$$F = \begin{bmatrix} F_W & 0 & 0 \\ 0 & F_L & F_V \end{bmatrix}$$

F matrix is estimated using the assumption that upwash and sidewash introduces perturbations in sideslip angle and angle of attack of follower aircraft. Rolling moment is directly added into moment equations.

and 'u_{wake}' vector dictates the wake model,

$$u_{wake} = \begin{bmatrix} \bar{W}_{wake}(y,z) & L_{wake}(y,z) & V^{CL}_{wake}(y,z) \end{bmatrix}^T$$

Three aspects must be considered when developing formation hold control system of the control system. First of all, the research focuses on an aircraft with relaxed static stability, hence a Stability Augmentation System (SAS) is implemented. In the outer loop, velocity and altitude hold control is developed. Finally a formation hold control system is developed and implemented to maintain a relative velocity and position between the follower and the leader aircraft in accord to the optimization of wake model.

E. Optimum Separation

Development of wake distribution model allowed us to calculate optimum separation between leader and follower aircraft at which drag is minimum.

Optimum separation was determined by calculating separation at the point of maximum thrust reduction, due to maximum upwash.

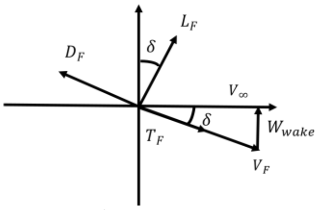

Fig. 3. Forces on Follower Aircraft in Formation

$$\frac{\Delta D}{D} = \frac{\Delta T}{T_0} = \frac{T_f - T_0}{T_0} \approx \frac{W}{T_0}\left(\frac{W_{wake}}{V_\infty}\right)$$

The distribution of drag reduction over the separation regime was modelled as follows:

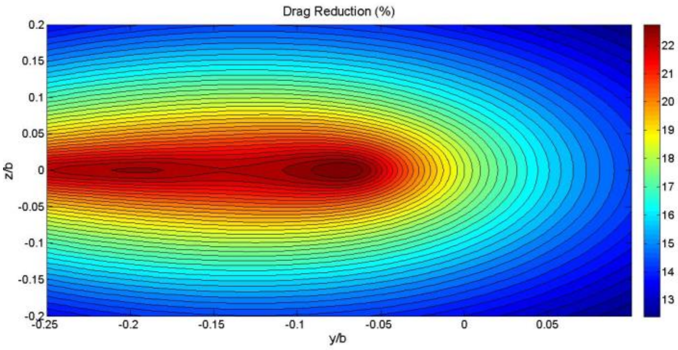

Fig. 4. Drag Reduction Distribution

Optimum Separation was calculated as follows:

$$x/b = 2$$
$$y/b = -0.072$$
$$z/b = 0$$

VII. Stability and Control Augmentation System

A. Control Scheme

The reference aircraft has relaxed longitudinal stability therefore, a control system was developed to stabilize the aircraft as well as provide altitude and velocity hold capability.

LQI control scheme was developed. A full state feedback along with integrated differences between reference and error of velocity, altitude and pitch angle were taken, and applied to the LQI gain (Fig. 6.)

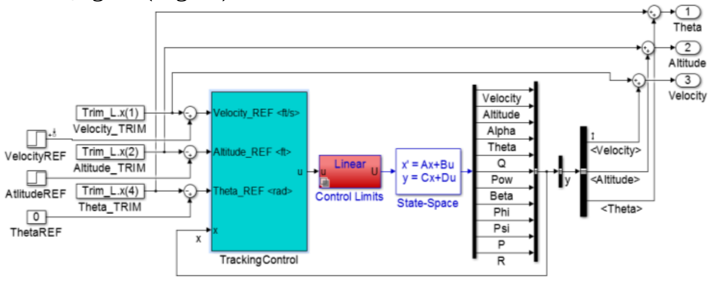

Fig. 5. Attitude Hold and SAS Control system

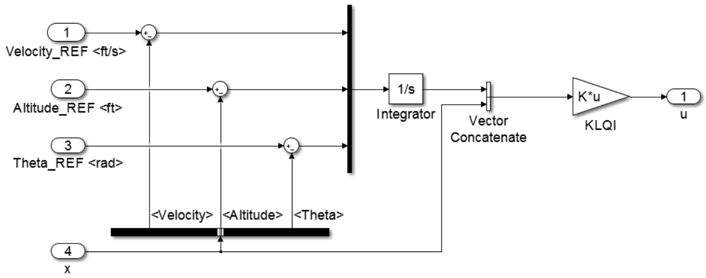

Fig. 6. Tracking Control Subsystem

B. Simulation Results

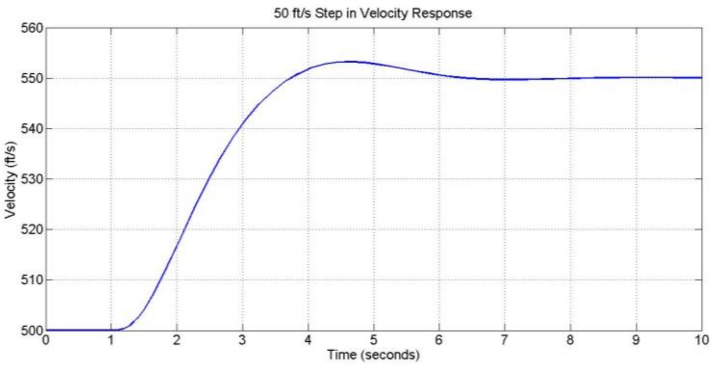

Fig. 7. Velocity Step Response

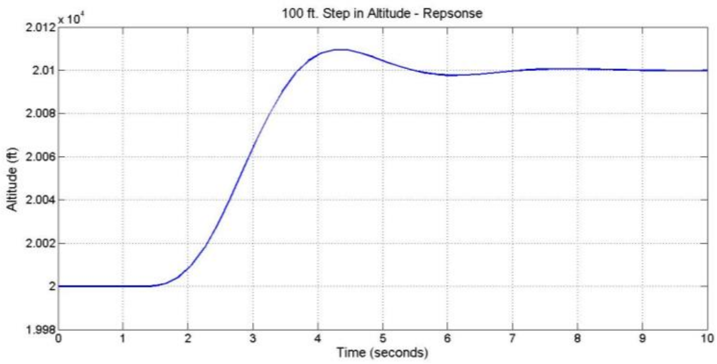

Fig. 8. Altitude Step Response

Fig. 7 and 8 show simulation results of the stability and control augmentation system. Fig. 7 shows the response when a 50 ft/s step in velocity is given to the controller, similarly Fig. 8 shows the response when a 100 ft. step in altitude is given to the controller. The results show that controller follows the step commands quite adequately and is valid for the reference aircraft.

VIII. Formation Hold Control System

A. Control Scheme

This control system enables the follower aircraft to maintain an optimum relative position and regulate relative velocity between two aircraft. The inner loop tracks and matches the velocity and altitude of follower aircraft with the leader aircraft using LQ based state feedback. The outer loop uses PID control scheme to maintain a desired relative position.

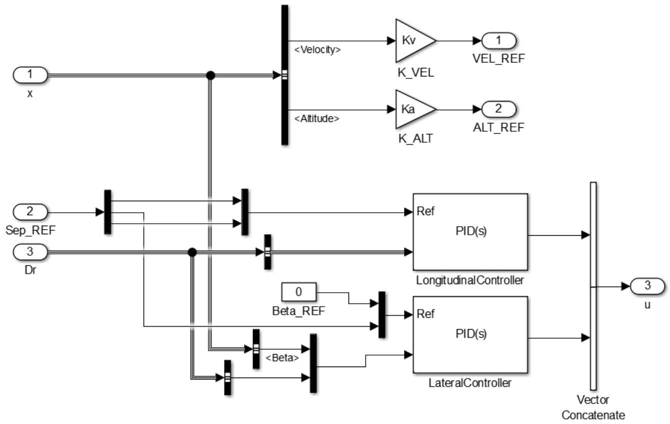

Fig. 9. Formation Hold Control

B. Simulation Results

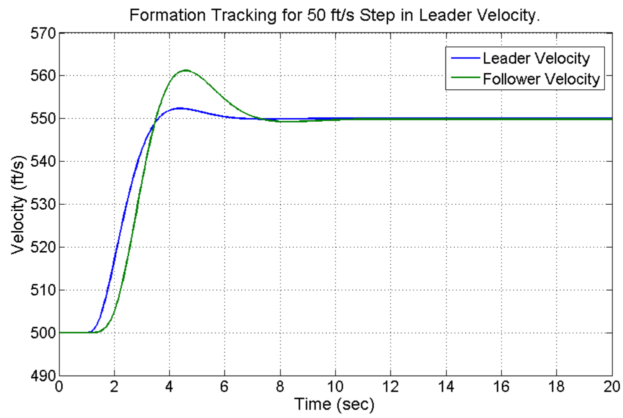

Fig. 10. Formation Hold Control Simulation

Fig. 10 shows simulation results of how the controller allows follower aircraft to track the leader aircraft's velocity in order to maintain a desired separation. The results show that follower aircraft fully matches the velocity of leader aircraft after a step of 50 ft/s is introduced in the leader aircraft. This simulation result validates this formation hold controller for the reference aircraft.

IX. Conclusion

This research resulted in development of a formation hold control system, which allows aircraft to maintain an optimum separation respect to each other. The optimum separation was calculated by developing a wake model distribution and analyzing drag reduction over a separation regime. The control systems were validated using simulation results.

X. Future Prospects

Formation hold control systems are slowly gaining popularity amidst the reliable autopilot systems available for aircraft in this and the coming era. This research is a stepping stone towards unlocking the full potential of unlocking automation formation flights.

Some of the future prospects of this project are; Improvement in wake model accuracy using VLM, hardware Implementation of the project, multi-aircraft formation control, formation configuration changing control design, nonlinear adaptive formation flight control.

References

[1] M. Pachter, J. D'Azzo, and J. L. Dargan, "Automatic formation flight control," *Journal of Guidance, Control, and Dynamics,* vol. 17, pp. 1380-1383, 1994/11/01 1994.

[2] M. Chiaramonti, F. Giulietti, and G. Mengali, "Formation Control Laws for Autonomous Flight Vehicles," in *Mediterranean Conference on Control and Automation*, 2006, pp. 1-5.

[3] Z. Chao, L. Ming, Z. Shaolei, and Z. Wenguang, "Collision-Free UAV Formation Flight Control Based on Nonlinear MPC," in *International Conference on Electronics, Communications and Control (ICECC)*, 2011, pp. 1951-1956.

[4] L. E. Buzogany, "Automated control of aircraft in formation flight," Master Thesis, Air Force Institute of Technology, 1992.

[5] M. J. Veth, "Advanced Formation Flight Control," Master Thesis, Air Force Institute of Technology, 1994.

[6] A. W. Proud, "Close Formation Flight Control," Master Thesis, Air Force Institute of Technology, 1999.

[7] P. Binetti, K. B. Ariyur, M. Krstic, and F. Bernelli, "Formation Flight Optimization Using Extremum Seeking Feedback," *Journal of Guidance, Control, and Dynamics,* vol. 26, pp. 132-142, 2003/01/01 2003.

[8] S. Ariyibi, "Nonlinear Control of Unmanned Aircraft Formations," Master Thesis, Middle East Technical University, 2014.

[9] L. T. Nguyen, M. E. Ogburn, W. P. Gilbert, K. S. Kibler, P. W. Brown, and P. L. Deal, "Simulator Study of Stall/Post-Stall Characteristics of a Fighter Airplane with Relaxed Longitudinal Static Stability.," NASA Langley Research Center1979.

[10] B. L. Stevens and F. L. Lewis, "Aircraft control and simulation, 1992," *John Wiley.*

[11] L. Sonneveldt, Q. P. Chu, and J. A. Mulder, "Nonlinear Flight Control Design Using Constrained Adaptive Backstepping," *Journal of Guidance, Control, and Dynamics,* vol. 30, pp. 322-336, 2007/03/01 2007.

[12] Y. Huo, "Model of F-16 Fighter Aircraft," 2007.

[13] B. L. Stevens and F. L. Lewis, *Aircraft Control and Simulation*: John Wiley, 1992.

[14] D. Burnham and J. Hallock, "Chicago Monostatic Acoustic Vortex Sensing System. Volume IV. Wake Vortex Decay," DTIC Document1982.

[15] J. D. Anderson Jr, *Fundamentals of aerodynamics*: Tata McGraw-Hill Education, 1985.

[16] G. Winckelmans, R. Cocle, L. Dufresne, and R. Capart, "Vortex methods and their application to trailing wake vortex simulations," *Comptes Rendus Physique*, vol. 6, pp. 467-486, 5// 2005.

[17] T. Gerz, F. Holzäpfel, and D. Darracq, "Commercial aircraft wake vortices," *Progress in Aerospace Sciences*, vol. 38, pp. 181-208, 4// 2002.

[18] C. Liu, "Wake Vortex Encounter Analysis with Different Wake Vortex Models Using Vortex-Lattice Method," Master's Thesis, Delft University of Technology, 2007.

APPENDIX

A. Inertia Estimation

In order to develop a mass varying dynamical model of the reference aircraft, variation in inertia tensor due to fuel consumption is required. This section focuses on estimating change in inertia due to fuel based on mass, geometry and location of fuel tanks present in the aircraft. It is assumed that fuel is being consumed from each tank simultaneously such that mass symmetry about XZ plane is conserved. Volumetric scaling was used for these calculations.

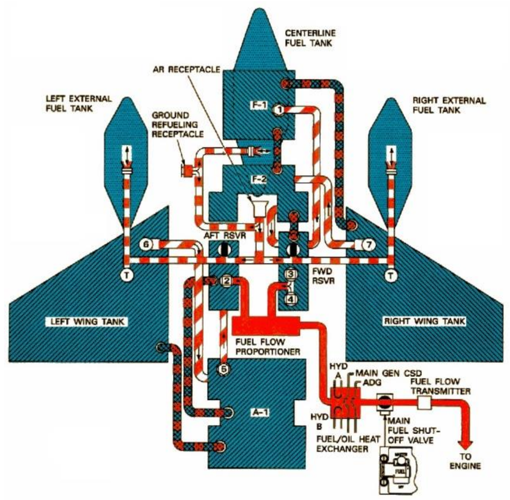

Fig. 11. Fuel tank schematics (Extracted from Flight Manual F-16C/D Block 50 LOCKHEED MARTIN CORPORATION 1996)

			C		D	
TANK LOCATION	FUEL QTY SEL KNOB SET-TINGS	POINTER	FUEL QTY (LB) JP-4	FUEL QTY (LB) JP-5/8	FUEL QTY (LB) JP-4	FUEL QTY (LB) JP-5/8
1. LEFT INTERNAL WING	INT WING	AL	525±100	550±100	525±100	550±100
2. RIGHT INTERNAL WING	INT WING	FR	525±100	550±100	525±100	550±100
3. F-1 FUSELAGE 4. F-2 FUSELAGE 5. FWD RESERVOIR	NORM	FR	3100±100	3250±100	1800±100	1890±100
6. AFT RESERVOIR 7. A-1 FUSELAGE	NORM	AL	2675±100	2810±100	2675±100	2810±100
5. FWD RESERVOIR	RSVR	FR	460±30	480±30	460±30	480±30
6. AFT RESERVOIR	RSVR	AL	460±30	480±30	460±30	480±30
8. CENTERLINE	EXT CTR	FR	1800±100	1890±100	1800±100	1890±100
9. LEFT EXTERNAL WING 370/600	EXT WING	AL	2300/3750±100	2420/3925±100	2300/3750±100	2420/3925±100
10. RIGHT EXTERNAL WING 370/600	EXT WING	FR	2300/3750±100	2420/3925±100	2300/3750±100	2420/3925±100
TOTAL INTERNAL FUEL			6625±300	6950±300	5650±300	5930±300
TOTAL EXTERNAL FUEL (370/600)			6400/9300±300	6730/9740±300	6400/9300±300	6730/9740±300

Fig. 12. Fuel tank schematics (Extracted from Flight Manual F-16C/D Block 50 LOCKHEED MARTIN CORPORATION 1996)