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An Efficient Variational Model for Vector Valued Image Segmentation

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Abstract*— Segmentation of images is a great challenge in the field of computer vision. Many variational models have been developed for this task such as Mumford Shah (MS) model and Chan Vese (CV) model. *Recently we have designed a model for gray images. But in that work, we employ a fresh method for vector-valued images for accurate segmentation. Test results are compared with state of the art model which validate robust performance of our proposed model.

Keywords—Image segmentation, Level set method, Variation al models, Euler's Lagrange equation, Numerical method.

I. Introduction

One of the most important and ubiquitous tasks in the field of image analysis is segmentation. The main aim of image segmentation is to partition a gray or color image into different sub-regions of homogeneous intensities such as texture, image segmentation is to partition a gray or color image into different sub-regions of homogeneous intensities such as texture, color etc. The main challenge of this problem is how to construct effective algorithms and methods to solve such problems. A number of image segmentation techniques is developed over the years, among those, active contour models got great attention since the proposal of a first active contour model by Kass et al. [3]. Active contour models are mainly organized into two groups: edge-based [1- 2] and region-based [3-4]. Edge-based models fit on edge information to stop the curve evolution. Edgebased models are better in images having strong object boundaries. However, these models have some drawbacks such as: high dependence on the initial position of the curve and are sensitive to noise and outliers. Whereas, region-based models [1-2], use region information such as texture, color, intensity etc. Region based models have no dependence on the initial position of the curve, less sensitive to noise and outliers and works well on images having weak object boundaries. Chan-Vese (CV) model [4] derived from the well-known Mumford-Shah (MS) functional [1] is a popular region-based model. The energy functional in term of the level set [7] formulation of their proposed model is given by :

$$F(u,C)=\mu.\int_C ds+\alpha\int_\Omega |u_0-u|^2\,dx+\beta\int_{\Omega/C}|\nabla u|^2 dx\,. \quad (1)$$

The first term of eq.(1) is the length term and its work is to bind the edge function. The 2nd term is fidelity term and its work is to endorse the smooth image u to be close to the given image u_0. The third term is called regulizer term and its work is to keep u smooth in the region Ω/C or in other words to assure that u is differentiable in the region Ω/C. μ, α and β are +ve parameters.

Chan and Vese [4] proposed energy functional, which is a special case of the (MS) model [1]], the energy functional of the CV model for vector-valued images is give

$$F(C,a_l,b_l)=\mu Length(C)$$
$$+\int_{inside(C)}\frac{1}{N}\sum_{l=1}^{N}\lambda_l^+|u_{0,l}-a_l|^2\,dxdy$$
$$+\int_{outside(C)}\frac{1}{N}\sum_{l=1}^{N}\lambda_l^-|u_{0,l}-b_l|^2\,dxdy\,, \quad (2)$$

where $\mu\geq 0$, $\nu\geq 0$, $\lambda_l^+>0$, $\lambda_l^->0$ positive parameters for each channel that can be chosen accordingly.

The CV model is expresse for images having intensity homogeneity as a result it does not work in those images having intensity inhomogeneity.

2. Proposed Model

The basic idea behind our proposed model is designed in the following way:

$u_0 = \beta J + N$,

where β is the intensity inhomogeneity , J is the ideal piecewise constant image and N is the additive noise. If the signal to noise ratio of u_0 is not too low. Clearly we have

$$\iota \log u_0 = \iota \log \beta J + \iota \log N. \quad (4)$$

To estimate the intensity inhomogeneity in small regions, we build a multi-scale average filter, based on the image local information we can write:

$$u_s = \frac{u_0 * K}{1 * K} \; , \quad (5)$$

$$u_D = \frac{u_s * K}{1 * K} \; . \quad (6)$$

It is understandable that u_D define the intensity distribution in local neighborhoods .

$$\iota o \mathrm{g}\, J^{\wedge} = \iota o \mathrm{g}\, u_0 - \iota \log u_D + \iota \log C_N . \quad (7)$$

the straightforwardness we set Eq.(7) as:

$$J^{\wedge} = \frac{C_N u_0}{u_D}. \quad (8)$$

The image $J^{\wedge} = \frac{C_N u_0}{u_D}$ is free of intensity inhomogeneity.

The energy functional in term of the level set formulation of our proposed model is given by:

$$F^{KBA}(a_1, b_1, \phi) = \int_{\Omega} \frac{1}{N} \sum_{l=1}^{N} \lambda_l^{+} (J^{\wedge} - a_1)^2 \phi H_c(1+\phi) dxdy$$
$$+ \int_{\Omega} \frac{1}{N} \sum_{l=1}^{N} \lambda_l^{-} (J^{\wedge} - b_l)^2 \phi H_c(1-\phi) dxdy \quad (9)$$

$$H_c(z) = \frac{1+z}{2}$$

$$+ \int_{\Omega} \frac{1}{N} \sum_{l=1}^{N} \lambda_l^{-} (J^{\wedge} - b_l)^2 \phi H_c(1-\phi) dxdy \quad (9)$$

$$H_c(z) = \frac{1+z}{2}$$

where u_D is the double filter formulation and C_N is the mean intensity of the given image $J^{\wedge}$. The constants $a_l(\phi)$ and $b_l(\phi)$ are defined as:

$$a_l(\phi) = \frac{\int_{\Omega} J^{\wedge} \phi H_c(1+\phi) dxdy}{\int_{\Omega} \phi H_c(1+\phi) dxdy} \quad (10)$$

$$b_l(\phi) = \frac{\int_{\Omega} J^{\wedge} \phi (1 - H_c(\phi)) dxdy}{\int_{\Omega} \phi (1 - H_c(\phi)) dxdy}. \quad (11)$$

The gradient descent formulation of over proposed model is given by:

$$\frac{\partial \phi}{\partial t} = -\delta_{\varepsilon}(\phi) \left[\begin{array}{l} \frac{1}{N} \sum_{l=1}^{N} \lambda_l^{+} (J^{\wedge} - a_l)^2 \\ + \frac{1}{N} \sum_{l=1}^{N} \lambda_l^{-} \left(J^{\wedge} - a_l \right)^2 \phi \end{array} \right]$$
$$- \left[\begin{array}{l} \frac{1}{N} \sum_{l=1}^{N} \lambda_l^{+} (J^{\wedge} - a_l)^2 \\ - \frac{1}{N} \sum_{l=1}^{N} \lambda_l^{-} \left(J^{\wedge} - b_l \right)^2 + \mu div(\frac{\nabla \phi}{|\nabla \phi} |) \end{array} \right]$$

3. Experimental Results

In this section we present some experimental resuls of Vector-Valued Khan-Badshah-Ali VVKBA model and Vector-Valued Chan-Vese VVCV model.

1. *Segmentation result of VVKBA and VVCV models for those images having intensity inhomogeneity.*

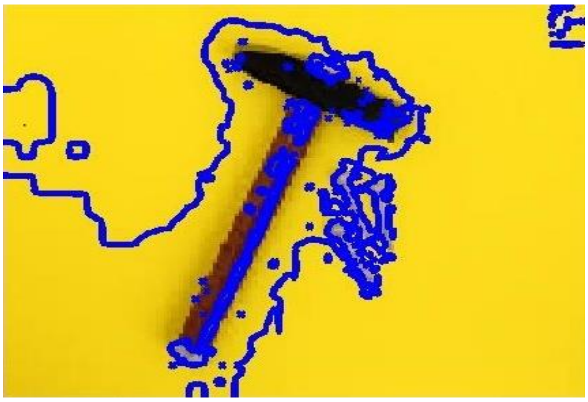
In Fig.1, is an RGB image with intensity inhomogeneity VVCV model does not segment those RGB image having intensity inhomogeneity. The performance of VVKBA model is better than VVCV model.

2. Segmentation result of VVKBA and VVCV models in multi-objects images

In Fig. 2, 3, 4 is an RGB images having intensity inhomogeneity. VVCV model does not segment those

RGB images having intensity inhomogeneity. The performance of (VVKBA) model is better than VVCV model.

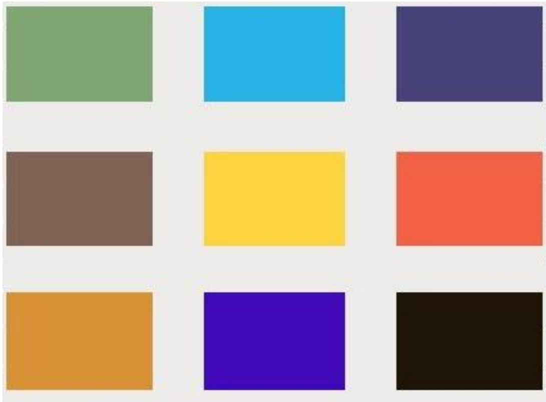
Given image



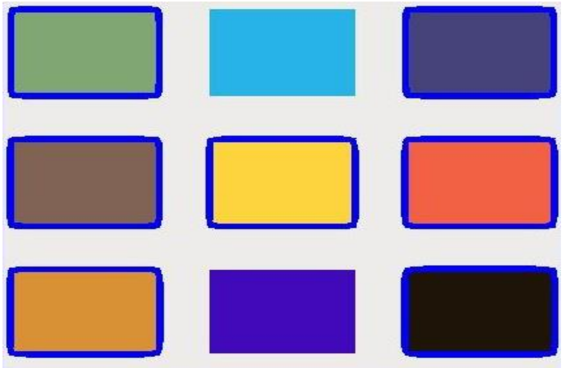
Result of VVCV

Result of VVKBA

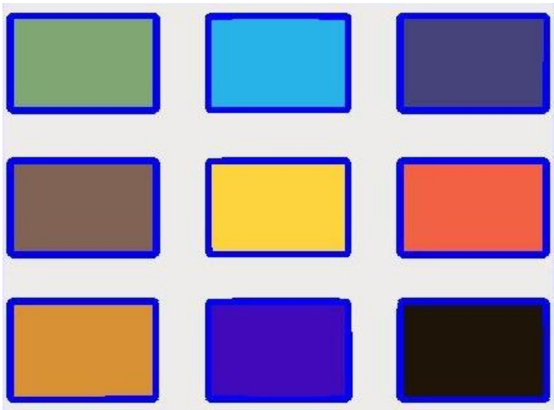
Segmentation result of vector-valued Chen-Vese model (VVCV) and our proposed vector-valued Khan-Badshah-Ali model (VVKBA): Parameter used for our proposed model are: $\sigma = 10,\ \lambda_1 = 1,\ \lambda_2 = 2,\ \ iter = 750.$



Given image



Result of VVCV



Result of VVKBA

Segmentation result of vector-valued Chen-Vese model (VVCV) and our proposed vector-valued Khan-Badshah-Ali model (VVKBA): Parameter used for our proposed model are: $\sigma = 10,\ \lambda_1 = 1,\ \lambda_2 = 1,\ \ iter = 750.$

Given image

Result of VVCV

Result of VVKBA

Segmentation result of vector-valued Chen-Vese model (VVCV) and our proposed vector-valued Khan-Badshah-Ali model (VVKBA): Parameter used for our proposed model are: $\sigma = 14,\ \lambda_1 = 1,\ \lambda_2 = 2,\ \ iter = 200.$

Given image

Result of VVCV

Result of VVKBA

Segmentation result of vector-valued Chen-Vese model (VVCV) and our proposed vector-valued Khan-Badshah-Ali model (VVKBA): Parameter used for our proposed model are: $\sigma = 24,\ \lambda_1 = 2,\ \lambda_2 = 1,$ $iter = 2000.$

4. Conclusion

In that work, we employ a fresh method for segmenting vector-valued images efficiently. We employed the dual filter formulation to handle intensity inhomogeneity .

The experimental results of our proposed model on synthetic images show efficient and robust performance.

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