

Comparison of Adaptive Beamforming Algorithms Robust Against Directional of Arrival Mismatch

Zafar Ullah Khan, A. Naveed, I. M. Qureshi, and Fawad Zaman

Abstract—The conventional adaptive beamformers face the problem of performance degradation in the presence of desired signal in the training data snapshots if there is a small mismatch between the presumed and the actual signal direction. In case of such mismatch, these beamformers generate the main beam in the presumed direction of desired signal and the actual signal with mismatched direction is considered as the interference, so the desired signal cancellation appears in this situation. This paper presents the effect of direction of arrival mismatch on traditional beamformers like minimum variance distortionless response (MVDR) and generalized sidelobe canceller (GSC). Furthermore, the performance comparison of different adaptive Beamforming algorithms which are robust against direction of arrival mismatch has also been done. These algorithms include worst case performance optimization beamformer for direction of arrival mismatch and generalized sidelobe canceller with modified blocking matrix. Simulation results show that the GSC with modified blocking matrix has better performance as compare to the worst case performance optimization beamformer for direction of arrival mismatch.

Index Terms— Adaptive beamforming, Minimum variance distortionless response (MVDR) beamformer, Generalized sidelobe canceller (GSC).

I. INTRODUCTION

WHEN exact knowledge of direction of arrival of desired signal and interferences is available, array signal processing with progressive phase can be used to generate the main beam along the desired signal direction and nulls in the direction of interferences [1]. On the other hand, adaptive beamforming, which has its applications in the field of radar, sonar, and wireless communications [2]-[4], requires only the desired signal direction. Minimum variance distortionless response (MVDR), sample matrix inversion (S MI), linearly constrained minimum variance (LCMV), and generalized sidelobe canceller (GSC) are the popular adaptive beamformers. These beamformers face the problem of performance degradation when the desired signal appears in the data snapshots and there is a mismatch between the

presumed and actual signal direction. Efforts are in progress to develop robust algorithms for such mismatches.

Diagonal loading [5] is a popular technique robust against direction of arrival mismatch but has no suitable way to find the diagonal loading factor. Another attractive approach is robust adaptive beamforming using worst case performance optimization [6]. But its performance is quite close to the simple algorithm known as diagonal loading of the sample matrix inversion (LSMI) algorithm.

In [7] a technique is presented which makes GSC robust against direction of arrival (DOA) mismatch by modifying its blocking matrix in order to broad the sharp nulls using the method discussed in [8].

This paper presents the performance comparison of different robust adaptive beamforming algorithms and utilizes Convex optimization as an alternative and efficient method to solve the null broadening problem for robust GSC [7], [8].

II. BACKGROUND

A. MVDR Beamformer

This beamformer minimizes the output power and maintains the distortionless response in the desired signal direction. Let $a(\theta_s)$ be the steering vector in the desired signal direction and R is the received signal covariance matrix, the optimization problem and its solution in terms of beamformer weight vector w_{MVDR} are given in (1) and (2) respectively.

$$\min_w w^H R w \quad \text{Subject to}$$

$$w^H a(\theta_s) = 1 \quad (1)$$

$$w_{MVDR} = \frac{R^{-1} a(\theta_s)}{a^H(\theta_s) R^{-1} a(\theta_s)} \quad (2)$$

Manuscript received May 29, 2012.

Received after re-formatting June 1, 2012

Received after modification as per reviewer's comments July 12, 2012.

Z. U. Khan and F. Zaman are with Department of Electronic Engineering, Faculty of Engineering and Technology, International Islamic University Islamabad, Pakistan. Phone:+923065482263; (e-mail: zafarullah.phdee13@iiu.edu.pk; fawad.phdee31@iiu.edu.pk).

A. Naveed is with School of Engineering & Applied Sciences (SEAS) Isra University, Islamabad Campus, Pakistan, (e-mail: anaveed@isra.edu.pk).

I. M. Qureshi is with Department of Electrical Engineering Air University Islamabad, Pakistan, (e-mail: imqureshi@mail.au.edu.pk).

B. SMI Beamformer

This beamformer minimizes the array output signal to interference-plus-noise ratio (SINR). Let R_s be the signal covariance matrix, the optimization problem for this beamformer is stated as:

$$\min_w w^H R w \quad \text{Subject to} \quad w^H R_s w = 1 \quad (3)$$

The solution to this optimization problem as given in [9] is $w_{opt} = P\{R^{-1}R_s\}$ where $P\{\cdot\}$ is the Eigen vector corresponding to the maximal Eigen value and w_{opt} is the weight vector.

C. Generalized Sidelobe Canceller (GSC)

GSC consists of two branches as shown in fig. 1. The upper branch with weight vector w_q is quiescent beamformer. The vector w_q preserves the desired signal in the upper branch. The lower branch is sidelobe cancelling branch which consists of blocking matrix B and adaptive weight vector w_a . It can be found from literature that $w_q = C(C^H C)^{-1}f$, $B = null\{C^H\}$, where C is the constraint matrix containing steering vectors corresponding to each constraint and f is gain vector containing gains corresponding to the constraints. The optimized adaptive weight vector w_a , in the lower branch, denoted by w_{ao} is given below in (4).

$$w_{ao} = (B^H R B)^{-1} B^H R w_q \quad (4)$$

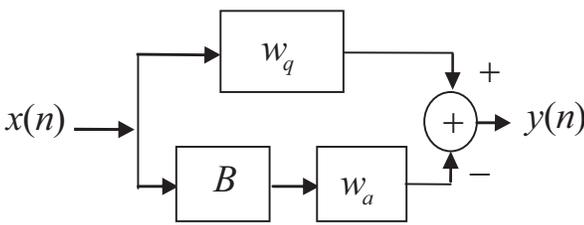


Fig. 1. Generalized Sidelobe Canceller

III. DOA MISMATCH EFFECT ON TRADITIONAL BEAMFORMERS

In order to observe the effect of direction of arrival mismatch on traditional beamformers, we have considered one desired signal with presumed direction along 0° and two interferences at 30° and 60° respectively.

A. Performance of MVDR Beamformer

Fig. 2(a) shows the performance of MVDR beamformer without mismatch i.e. presumed and actual signal are in the

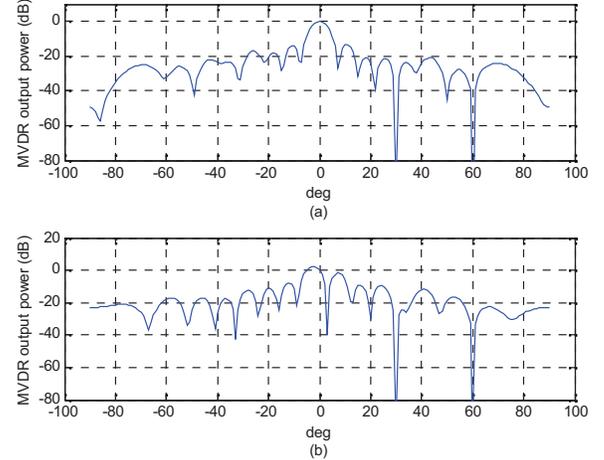


Fig. 2. . (a) MVDR beamformer output without mismatch; desired signal is at 0° . The interferences are at 30° and 60° . (b) Output of MVDR beamformer with mismatch when presumed signal is at 0° while its actual direction is along 3° . The interferences are same as in (a).

same direction. Fig. 2(b) shows performance degradation of this beamformer when actual signal direction is at 3° and presumed direction is along 0° , i.e. for a mismatch of 3° .

B. Performance of SMI Beamformer

Fig. 3(a) and 3(b) show the performance of SMI beamformer without and with mismatch respectively.

C. Performance of Generalized Sidelobe Canceller (GSC)

The performance of GSC without and with mismatch has been shown in fig. 4(a) and 4(b) respectively.

IV. ROBUST ADAPTIVE BEAMFORMERS

In this section two robust beamforming algorithms i.e. worst case performance optimization (WCO) beamformer and robust GSC for direction of arrival mismatch are being discussed.

A. Worst Case Performance Optimization (WCO) Beamformer

Assume that Δ is the error matrix in R_s due to mismatch in desired signal direction. Let Δ be bounded by some known positive constant ε i.e. $\|\Delta\| \leq \varepsilon$. Where $\|\cdot\|$ denotes Frobenius norm of a matrix. In [9], the constraint for SINR maximization has been modified for the beamformer robustness against direction of arrival mismatch as given below

$$w^H (R_s + \Delta) w \geq 1 \quad \text{For all } \|\Delta\| \leq \varepsilon$$

So SMI optimization problem becomes as

$$\min_w w^H R w \quad \text{Subject to}$$

$$w^H (R_s + \Delta) w \geq 1, \quad \text{for all } \|\Delta\| \leq \varepsilon \quad (5)$$

For the worst case performance, Δ can be found by solving the following optimization problem

$$\min_{\Delta} w^H (R_s + \Delta) w \text{ Subject to } \|\Delta\| \leq \varepsilon.$$

For this problem Δ comes out to be [9]

$$\Delta = -\varepsilon \frac{w w^H}{\|w\|^2} \quad (6)$$

By putting the value of Δ , the optimization problem (5) becomes as

$$\min_w w^H R w \text{ Subject to } w^H (R_s - \varepsilon I) w = 1$$

The optimum weight vector for the robust beamformer comes out to be

$$w_{rob} = P\{R^{-1}(R_s - \varepsilon I)\} \quad (7)$$

To overcome other array imperfections, similar mismatch Δ_1 is applied in R , with the condition $\|\Delta_1\| \leq \gamma$. The robust weight vector, as given in [9], comes out to be

$$w_{rob,s} = P\{(R + \gamma I)^{-1}(R_s - \varepsilon I)\} \quad (8)$$

B. Proposed solution for Robust GSC

Blocking matrix B in the lower branch of GSC, in fig. 1, blocks the desired signal by producing sharp nulls in the presumed signal direction. In case of no mismatch, the desired signal falls in these sharp nulls and is blocked in the lower branch. In the presence of mismatch, the desired signal falls outside these sharp nulls and appears in the lower branch and is cancelled when the outputs of two branches are subtracted. To overcome this problem, null broadening for the vectors of B was suggested in [7]. This null broadening method has been discussed in [8] and is given below for robustness of the GSC.

Let b_0 be a vector in matrix B and let b be the desired alternate vector with broad null then the optimization problem to find b can be stated as

$$\min_b (b - b_0)^H (b - b_0) \text{ Subject to } b^H Q b \leq \xi \quad (9)$$

where Q , a Hermitian matrix of order M , is used to broaden the sharp null at θ_0 and is given by

$$Q = \int_{\theta_0 - \Delta\theta/2}^{\theta_0 + \Delta\theta/2} a(\theta) a^H(\theta) d\theta$$

In the above expression, $\Delta\theta$ is the region or space with center at θ_0 over which broad null is required and $\xi \geq 0$ gives mean square null depth over the region $\Delta\theta$. In [7], and [8] a complicated method is used to solve (9).

We propose convex optimization technique to solve the expression (9) because it is convex optimization problem. It can be solved using a few commands of convex optimization software like [10].

V. SIMULATION RESULTS FOR ROBUST ADAPTIVE BEAMFORMERS

A uniform linear array of 16 antenna elements has been used with inter element spacing $\lambda/2$. One desired signal with presumed direction along 0° and two interferences at 30° and 60° respectively have been used for simulation in MATLAB. All the results have been averaged over 500 snapshots.

A. Performance of Worst Case Optimization Beamformer

Fig. 3(b) shows the performance of SMI beamformer for comparison with WCO for the 3° mismatch i.e. the actual signal direction is along 3° . Fig. 3(c) shows the performance of WCO beamformer for the same situation as in fig. 3(b). For fig. 3(c), we have used $\varepsilon = 5.3$ and $\gamma = 50$.

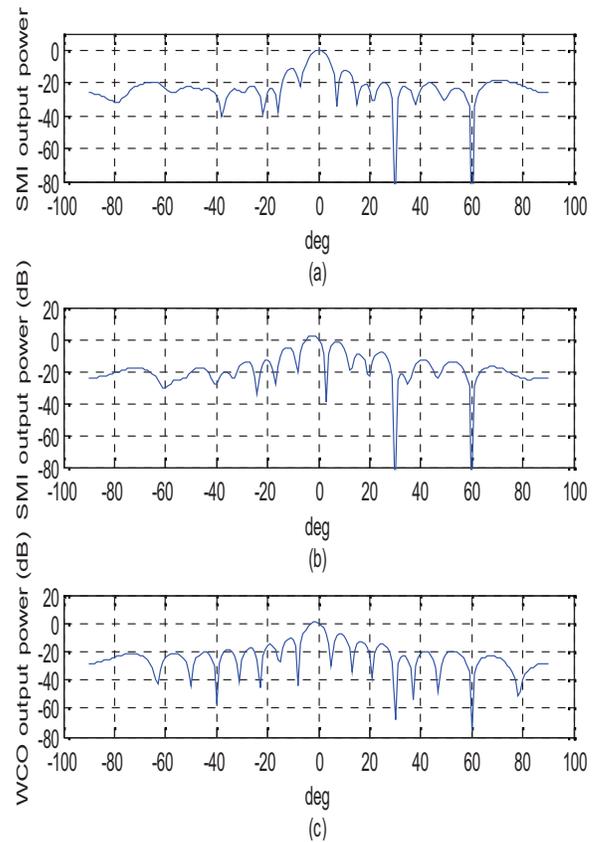


Fig. 3. (a) Output of SMI beamformer without mismatch; desired signal is at 0° . The interferences are at 30° and 60° . (b) SMI beamformer output power when desired signal is presumed at 0° but actually it is along 3° with the same interferences as in (a). (c) Output power of worst case optimization (WCO) beamformer for the same situation as in (b).

B. Performance of Robust GSC

Fig. 4(b) shows the performance of traditional GSC for comparison with robust GSC when the actual signal direction is along 3° . Fig. 4(c) shows the performance of robust GSC for the same situation as in fig. 4(b). The blocking matrix has been modified to broaden the nulls by 6° i.e. 3° on either side of the presumed signal direction.

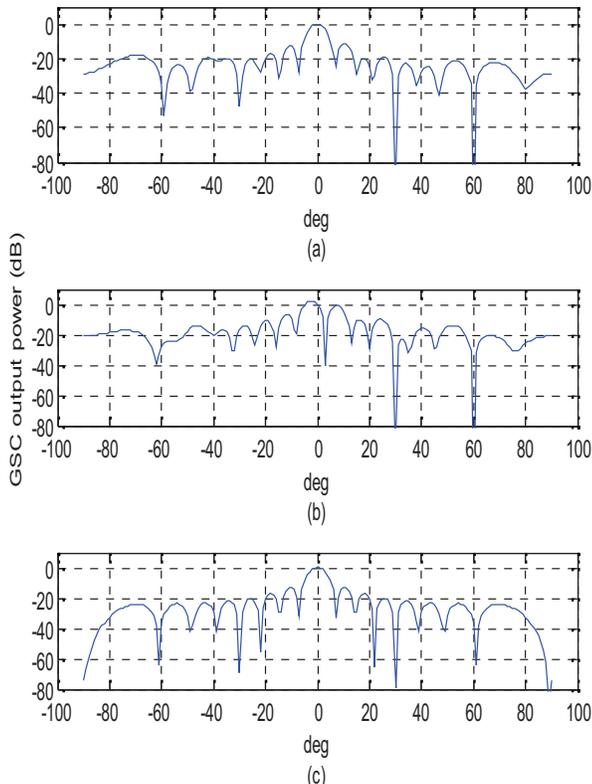


Fig. 4. (a) Output of GSC without mismatch when desired signal is at 0° . The interferences are at 30° and 60° . (b) Output of GSC with mismatch i.e. desired signal is presumed at 0° but its actual direction is along 3° with the same interferences as in (a). (c) Output power of robust GSC with modified blocking matrix for the same situation as in (b).

VI. RESULTS AND DISCUSSION

It can be seen from fig. 2, 3 and 4 that the traditional beamformers like MVDR, SMI and GSC face the problem of performance degradation when there is mismatch between the presumed and actual signal direction. The output power of these beamformers in the actual signal direction (at 3°) is -40 dB. The output power of worst case performance optimization beamformer at actual signal direction (at 3°) is approximately -7.2 dB, so there is great improvement in performance for this beamformer as compare to SMI beamformer (fig. 3(b)).

Fig. 4(c) shows the output power for robust GSC with modified blocking matrix. This pattern resembles the pattern for the traditional GSC without direction of arrival mismatch that is why fig. 4(c) and fig. 4(a) show similar performance. The output power of robust GSC, for mismatched signal, is

equal to -2.68 dB which is -40 dB for the traditional GSC of fig. 4(b).

The output power of the robust GSC and worst case optimization beamformer, for the mismatched signal, is equal -2.68 dB and -7.2 dB respectively. Clearly, the robust GSC has better performance. Another problem with worst case optimization beamformer is that it has no defined criterion to select diagonal loading parameters ϵ and γ .

VII. FUTURE WORK

A comprehensive study of modified blocking matrix of robust GSC, using the above mentioned null broadening technique, is required. Ideally the output power pattern of vectors of traditional and modified blocking matrix should be quite similar and the modified vectors should have broad nulls in the signal mismatch region as compared to traditional vectors. Some other null broadening techniques can also be used to guarantee the response close to ideal.

REFERENCES

- [1] Z. U. Khan, A. Naveed, I. M. Qureshi, F. Zaman, "Independent Null Steering by Decoupling Complex Weights", *IEICE Electronics Express*, vol.8, no. 13, pp. 1008-1013, July 10, 2011.
- [2] M. Bengtsson, and B. Ottersten, "Optimal downlink beamforming using Semidefinite optimization," In Proc. of 37th annual Allerton Conf. on Comm., Control, and Computing, pp. 987-996, Sep. 1999.
- [3] V. Sharma, I. Wajid, A. B. Gershman, H. Chen, and S. Lambotaran, "Robust Downlink Beamforming Using Positive Semi-definite covariance constraints," *IEEE International ITG Workshop on smart antennas (WSA 2008)*, pp. 36-41.
- [4] Brennan, L. E. J. D. Mallet, and I. S. Reed, "Adaptive Arrays in Airborne MTI radar," *IEEE Trans. Antennas Propagation*, vol. 24, pp. 607-615, Sept. 1976.
- [5] B. D. Carlson, "Covariance matrix estimation errors and diagonal loading in adaptive arrays," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 24, pp. 397-401, July 1988.
- [6] S. A. Vorobyov, A. B. Gershman, and Z. Q. Luo, "Robust Adaptive Beamforming Using Worst-Case Performance Optimization: A Solution to the Signal Mismatch Problem," *IEEE Trans. on Sig. processing*. Vol. 52, no. 2, pp. 313-324, Feb. 2003.
- [7] L. Lei, X. Rongqing, and L. Gaopeng, "Robust Adaptive Beamforming Based on Generalized Sidelobe Cancellation," CIE International Conference on Radar, Oct. 16-19, 2006, Shanghai, China.
- [8] M. H. ER, "Linear Antenna Array Pattern Synthesis with Prescribed Broad Nulls," *IEEE Trans. on Antenn. and Prop.*, vol. 38, No. 9, pp. 1496-148, Dec. 1990.
- [9] S. Shahbazpanahi, A. B. Gershman, Z.-Q. Luo and K. M. Wong, "Robust adaptive beamforming for general-rank signal models," *IEEE Trans. Signal Processing* (vol. 51), pp.2257-2269, Sept. 2003.
- [10] M. Grant, and S. Boyd, "CVX: A system for disciplined convex programming; cvx version 1.21" April, 2