Design and Computational Analysis of a Dual-Bell nozzle

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Dual-Bell nozzle belongs to the family of altitude adaptive advanced rocket nozzles. The precedence which Dual-Bell nozzle holds over a conventional nozzle is that while the later provides optimum performance at only one design altitude, the former is capable of generating optimum performance at two design altitudes. Dual-Bell nozzle comprises of three essential constituents; the base contour, wall inflection and the extension contour. It works optimally at sea level supplementing a fully expanded flow which increases thrust and avoids premature flow separation, hence, deterring the occurrence of side loads produced at low altitudes. Subsequently, the flow reattaches to the extension nozzle at the inflection point bringing the entire area ratio into utility and an increased performance in the vacuum is obtained by an enhanced vacuum thrust. It has no additional mechanical devices or moving parts, which is a major advantage it holds over other altitude adaptive nozzles.

The author uses the most optimum value of area ratio, nozzle length and inflection point. Rao's Parabolic Method is employed to design the parabolic contour for optimum thrust production. Propellant mass consumption for each Rao's nozzle is calculated. CFD analysis of the resultant flow field is carried out to verify the required critical flow transition, and the resultant effect on performance gains.

I. NOMENCLATURE

P	=	Pressure (Pa)
V	=	Velocity (m/s^2)
M	=	Mach No.
F	=	Thrust (N)
g	=	gravitational constant (m/s^2)
Isp	=	Specific impuse
AR	=	Area ratio
Θ_b	=	Inflection angle
Θ_e	=	Exit angle
k	=	Specific heat ratio
K	=	Percentage length of conical nozzle
ε	=	Expansion ratio

Subscript

b	=	base contour
e	=	extension contour

II. INTRODUCTION

Man's curiosity has compelled him to discover means to venture beyond the upper atmosphere and into the vastness of space. Thereafter, it is the aerospace engineering community's endeavor to invent and redefine efficient and cost effective means of transportation. This leads to an in depth research on rocket engine subsystems and finding their alternative, more robust configurations. One such way of minimizing the costs by saving fuel is to analyze and redesign the rocket nozzle.

A rocket works by Newton's third law of motion which states that every action has an equal but opposite reaction. [1] In accordance with this, the action is the expulsion of the propellant burnt in the combustion chamber, out through the nozzle's diverging section. As a result, thrust is produced which propels the rocket forth. During the rocket's flight the flow in the nozzle is subjected to three cases; under-expansion, optimal expansion and over-expansion. At lower altitudes the ambient pressure is greater than the exit pressure, the nozzle is over-expanded and the flow separates from inside the nozzle's wall before reaching the exit. This is a dangerous case, since it leads to the production of dangerous side loads which can cause vibrations within the rocket nozzle. At one point during a conventional nozzle's flight the ambient

Fig.1. A Dual-Bell nozzle.

pressure becomes equal to the exit pressure. This is the design point since the flow is optimally expanded, it separates from the nozzle's exit and gives maximum thrust. As the rocket progresses and the altitude increases, the ambient pressure becomes less than the exit pressure. This causes the nozzle to become under-expanded since the flow separates outside the nozzle's exit. This can lead to a loss of thrust since the kinetic energy is converted to thrust outside the nozzle.

A focus of modern nozzle research is the diverging section. A Dual-Bell nozzle is an altitude adaptive rocket nozzle, which works optimally at two design points. The first design point is optimized for lower altitudes to prevent under-expansion and hence, the generation of dangerous side loads. The second design point is designed for higher altitudes, which can stall under-expansion and give optimum thrust at higher altitudes. This concept is especially useful in single-stage-to-orbit applications. Since, as opposed to a multi-stage rocket where the stages separate at their respective altitudes, an SSTO (Single Stage to Orbit) launch vehicle has to cover the entire trajectory alone, this means that needs an exceedingly efficient nozzle which can overcome over-expansion quickly and delay under-expansion as far as possible. [2]

This Dual-Bell nozzle is designed for an altitude of 21000 m where the rocket powers off. Taking the entire trajectory into consideration, the design point is constructed at regular intervals and the performance parameters are calculated. A design area ratio for a lower altitude and one for a higher altitude is selected based on the propellant mass consumption and the performance of the rocket nozzle. The contour of the Dual-Bell nozzle are constructed using Rao's parabolic method. The nozzle is imported into ANSYS Workbench 14.5 and modelled in Design Modeler. The pressure and Mach contour graphs are constructed using ANSYS Fluent. The specific impulse of the Dual-Bell nozzle is compared to that of the Rao nozzle from which it has been constructed to evaluate the improvement in performance.
A considerable amount of literature is found on this prospective advanced nozzle technology, especially during the 2000's after the introduction of vigorous CFD techniques which can computationally analyze the design and evaluate performance leading to better judgments. It is hoped that the concept may come into real time application by 2020.

III. DESIGN METHODOLOGY

At every 0.1 of the total altitude which is 21000m, the ambient pressure has been taken equal to the exit pressure. The pressure ratios have been back solved to find the Mach No., the exit pressure and area ratio from which exit area of the nozzle is found. Henceforth, eleven Rao nozzles with distinct area ratios have been constructed. Thrust for the entire trajectory of each nozzle is plotted using MATLAB. Specific Impulse is calculated from the thrust at the design point of each nozzle. Rao' parabolic method is used to construct the nozzle. [3]

A. Determining Performance Parameters of Rao nozzles

The throat area, pressure and temperature is the same for all eleven Rao nozzles. The following equations are used to calculate the throat parameters:

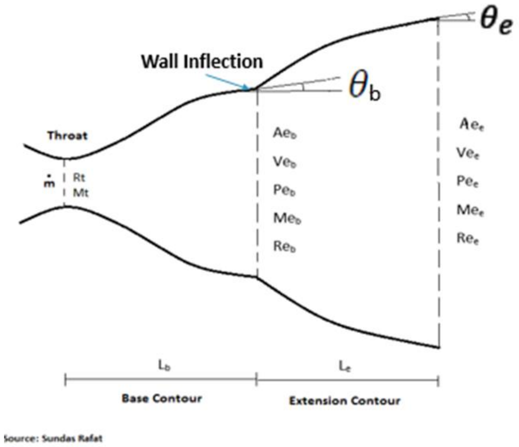

Fig.2. Base contour and extension contour parameters of a Dual-Bell nozzle labelled.

TABLE.1.

Rocket Performance Parameters	
Thrust Required (N)	15000
Isp Required (s^{-1})	250
Ratio of specific heats	1.23
Molar Mass (kg/mol)	28

Chamber Pressure (Pa)	500,00,00
Chamber Temperature (K)	3300
Characteristic Exhaust Velocity (m/s)	1400
Throat Mach No.	1
Gravitational constant (m/s^2)	9.8
Gas Constant (J/kg.K)	8.314*1000/Mw
Inert Mass of Rocket (kg)	3000
Mass of Propellant (kg)	7000
Burn Time (s)	560
Total Altitude (m)	21000
Density of Stainless Steel 302 (kg/m^2)	7861.09

Throat Area: $A_t = \frac{c^* \dot{m}}{P_c}$

Throat Radius: $R_t = \sqrt{A_t/\pi}$

Throat Pressure: $P_t = P_c[1 + \frac{k-1}{2}]^{-\frac{k}{k-1}}$

Throat Temperature: $T_t = T_c[\frac{1}{1+\frac{k-1}{2}}]$

Mass Flow Rate: $\dot{m} = \frac{F}{g_0 Isp}$

TABLE.2.

Throat Conditions (Calculated)	
Mass flow rate (kg/m^3)	6.1224
Throat Radius (m)	0.0234
Throat Area (m^2)	0.00171
Throat Pressure (Pa)	279,3500
Throat Temperature (K)	2959.6

To begin with the exit parameters of the base contour need to be determined in order to find the area ratio. The base contour is optimized for lower altitudes.

$$M_e^{\,2} = \frac{2}{k-1}\left[\left(\frac{P_c}{P_a}\right)^{\frac{k-1}{k}} - 1\right]$$

$$\varepsilon = \frac{A}{A*} = \left(\frac{k+1}{2}\right)^{\frac{-k+1}{2(k-1)}} \frac{\left(1+\frac{k-1}{2}M^2\right)^{\frac{k+1}{2(k-1)}}}{M}$$

$$A_e = \varepsilon A_t$$

$$v_e = \sqrt{\frac{2k}{k-1} R T_c \left(1 - \left(\frac{P_e}{P_c}\right)^{\frac{k-1}{k}}\right)}$$

$$T = \dot{m}\mathrm{Ve} + (\mathrm{Pe} - \mathrm{Pa})\mathrm{Ae}$$

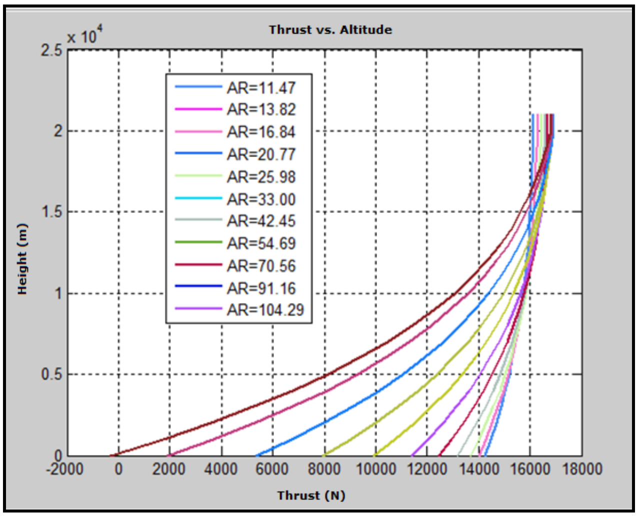

Fig.3. Thrust vs. altitude graph for eleven Rao nozzles.

The nozzle cannot be designed to operate optimally at sea-level because that would consume a great amount of fuel, which would significantly reduce the efficiency. [4] The optimum area ratio must lie somewhere between 13.82 and 16.84 which are optimized at 2.1 km and 4.2 km respectively. According to the propellant mass consumption graph, after an area ratio of 15 the propellant consumed by the rocket becomes relatively constant. Hence, the base contour is optimized at an altitude of 3.05 km and an area ratio of 15.

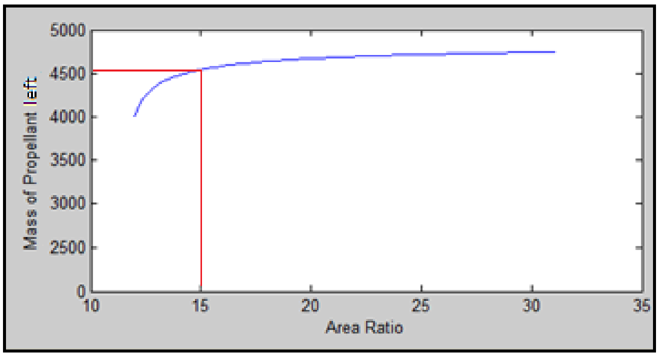

Fig.4. Propellant mass consumption at different area ratios.

The extension contour is designed to give optimum performance at a higher altitude which according to literature is approximately two-third of the entire altitude. [5] However, since the nozzle was once optimized at a lower altitude, under-expansion is slightly delayed, which means that we can select a higher point. Subsequently, the design point lies somewhere between 54.69 km and 70.56 km at 14.7 and 16.8 km respectively. A compromise is taken at an altitude of 15.5 km and the area ratio of 60 is taken for the extension contour.

TABLE.3.

	Area Ratio	Design Altitude (m)	Exit Pressure (P)	Exit Mach No.	Exit Velocity (m/s^2)	Thrust (N)	Isp
1	15	3050	5.3563e+04	3.2700	2.4236e+03	1.4751e+04	245.8500
2	60	15500	1.0628e+04	4.3325	2.6762e+03	1.6398e+04	273.3000

The specific impulse at an area ratio of 15 is calculated to be 245 and for an area ratio of 60 is 273 at design altitudes. Inorder to calculate the specific impulse at the end of a Dual-Bell nozzle at the design altitude the exit velocity needs to be determined. This can be done in ANSYS CFX.

B. Construction of Rao's nozzle

Rao's parabolic method is used to design the contour of the two Rao nozzles which have been selected and then the Dual-Bell nozzle using MATLAB. [6]

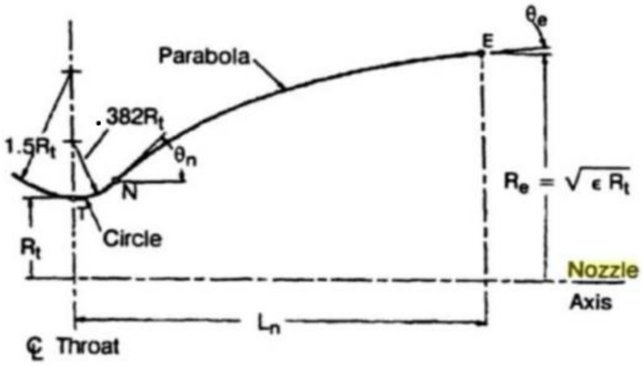

Fig.5. Rao's parabolic mathod. Source: Kulhanek, 2012.

Rao defined a thrust optimized parabolic nozzle by two circles of radius 1.5 R_t and 0.382 R_t respectively, and a parabola. The length of the nozzle is given by the formula:

$$L_b = \frac{K(\sqrt{\varepsilon}-1)R_t}{\tan(\theta_b)}$$

Where K is the percentage length of the conical nozzle of a 15° half angle is taken to be 0.8, ε is the area ratio, R_t is the throat radius and θ_b is the inflection angle.

The nozzle parameters have to be established using a coordinate system in order to construct it. The nozzle can be plotted by converting the equation of the circle in terms of the y-coordinate.

The equation of the first circle:

$$x^2 + (y - (R_t + 1.5R_t)^2) = (1.5R_t)^2$$

The equation of the second circle:

$$x^2 + (y - (R_t + 0.382R_t)^2) = (0.382R_t)^2$$

The equation of the parabola:

$$x^2 = ay^2 + by + c$$

Linear system if equation in matrix form:

$$\begin{bmatrix} 2R_N & 1 & 0 \\ 2R_e & 1 & 0 \\ R_N^2 & R_N & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \frac{1}{\tan(\theta_N)} \\ \frac{1}{\tan(\theta_e)} \\ x_N \end{bmatrix}$$

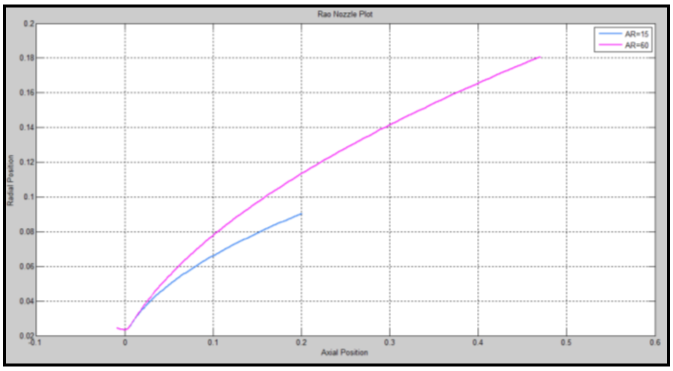

Fig.6. Rao nozzle plots: The blue line represents the Rao nozzle with an area ratio of 15 and the pink line represents an area ratio of 60.

The Dual-Bell nozzle is constructed in a similar fashion. The length of the second contour is determined using the same equation for length:

$$L_{\varepsilon} = \frac{K(\sqrt{\varepsilon}-1)R_t}{\tan(\theta_{\varepsilon})}$$

where θ_{ε} is the exit angle. The value of K is taken to be 0.7 in order to conserve the length of the nozzle.

The values of the coefficients are determined from the model formed from the linear system of equations.

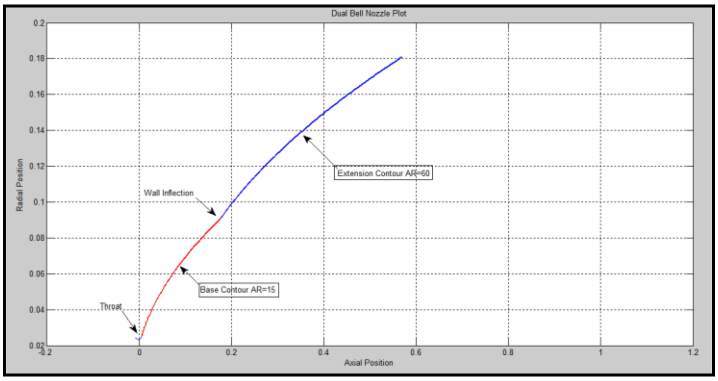

Fig.7. Dual-Bell nozzle with a base contour of AR=15 and an extension contour of AR=60.

IV. Analysis and Results

The Dual-Bell nozzle was modeled using ANSYS Design Modeler, meshed in Mesh Generator and analyzed in ANSYS CFX 14.5. The thrust and the velocity at the exit of the Dual-Bell nozzle are determined and compared to that of the conventional nozzle. The pressure and the Mach contour plots of the nozzle are extracted from the results in CFX.

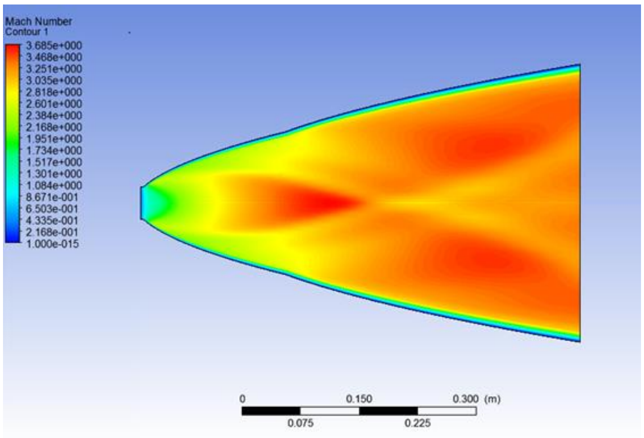

Fig.8. Mach contour of Dual-Bell nozzle given by ANSYS CFX 14.5 at design altitude.

The graphs show a presence of the formation of triple point inside the nozzle slightly offset from the wall inflection. The flow transitions from the throat to the base contour, from where it follows the nozzle's divergent section until it reaches the wall inflection. At the first design altitude the flow separates at the wall inflection. As the altitude increases the flow follows the contour of the extension contour and at the second design point separates from the extension contour's exit. [7]

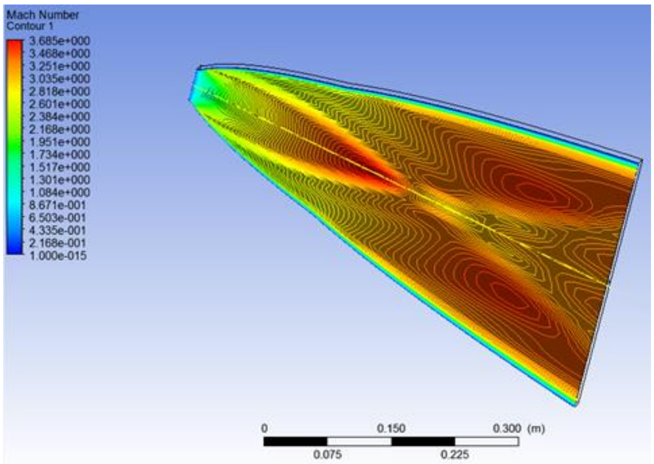

Fig.9. Mach contour of Dual-Bell nozzle with the flow characterictics.

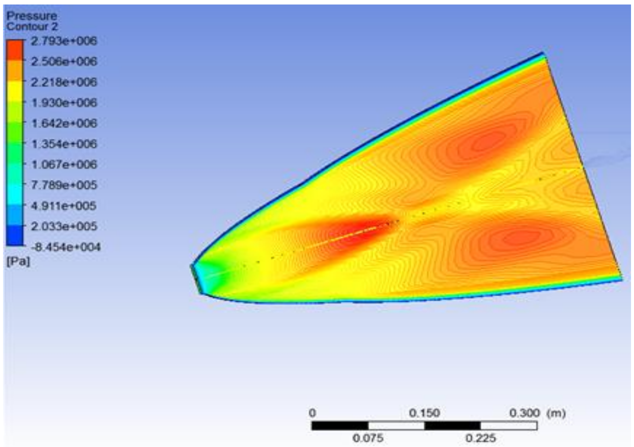

Fig.10. Pressure contour plots using ANSYS CFX at design altitude.

The thrust is calculated using the equation:

$$T = \dot{m}\mathrm{Ve} + (\mathrm{Pe} - \mathrm{Pa})\mathrm{Ae}$$

The specific Impulse is calculated using the formula:

$$I_{sp} = \frac{T}{\dot{m}g_o}$$

The specific Impulse of the Dual-Bell nozzle is calculated to be 293 s^{-1}. This indicates that the specific impulse of the Dual-Bell nozzle at the design altitude is more than that of the Rao nozzle it has been constructed from.

V. CONCLUSION AND RECOMMENDATION FOR FUTURE WORK

The Dual-Bell nozzle performs better than a conventional nozzle as can be seen from the results as it provides an 8.2% increase in the specific impulse which in turn leads to lesser consumption of fuel. The author suggests that the Dual-Bell nozzle's shape could be polished. This includes determining the shape of the inflection point in order to ensure the smooth transition of the flow from the base contour to the exit contour. The CFD (Computational Fluid Dynamics) analysis can be refined to observe the flow transition characteristics. Moreover, a variety of Dual-Bell nozzles can be formed and their performances analyzed, keeping the base contour constant and varying the area ratio and length of the extension contour. Subsequently, the area ratio of the extension contour can be fixed and the base contour area ratios for the lower altitudes can be varied. This will result in the production of eight nozzles, on which a detailed CFD analysis can be carried out. This will further lead to the proper verification of the results of the Dual-Bell nozzles performance as compared to a conventional nozzle.

VI. ACKNOWLEDGMENT

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VII. REFERENCES

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