Cumulative Damage Estimation Algorithm under the Application of FALSTAFF Spectrum

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Abstract— **Aluminum Alloy 2024 is commonly used in aircraft structure, hence to make an estimate of fatigue life is very important and should be accurate. This paper reviews the theoretical analysis of fatigue life of Aluminum Alloy Compact Tension Specimen using FALSTAFF (a variable amplitude loading spectrum). Stress Analysis was performed in ANSYS. FALSTAFF spectrum was generated in MATLAB using AFGROW directory. Rain-flow counting method was used to calculate the number of cycles corresponding to each stress magnitude. A simplified MATLAB code based algorithm was designed and verified to implement Rain-flow counting method. Total number of cycles to failure corresponding to each stress in the spectrum was taken from the S-N curve of Aluminum Alloy 2024. By comparison with the number of cycles in the spectrum, cumulative damage was calculated using Miner's Rule. Fatigue life of the component was then estimated by interpolation. These results were then compared against literature survey. After getting the results, proper suggestions and precautionary measures were given to keep the structure safe within safe limits.**

Keywords* — *Fatigue, FALSTAFF, Rain-flow, ASTM

I. INTRODUCTION

Life assessment of structural components is used to avoid catastrophic failure and to maintain safe and reliable functioning of equipment [1]. In structural technology, it is important to have appropriate knowledge of remaining life, fitness-for-service, inspection intervals, and reliability of structural components. Failure of structure results in the loss of lives in addition to cost of repair [1]. The consequences and cost of fractured components is unwanted. Hence prediction and estimation of fatigue life is important. Engineers are continuously trying to prevent reoccurrence of failure. Different studies have been conducted but still this is a very big issue to solve.

The main problem is fatigue phenomena which initiates crack in the structure and ultimately results in failure. A lot of accidents occurred in industry, due to this phenomenon [2]. It is critical because the structure fails before its maximum load bearing point. There are various factors that limit the structural life. Some common life limiting factors to most structures are material defects, fabrication practices, stress, stress concentration, temperature, corrosion concerns, and improper maintenance [3]. Different methods are present to predict number of load cycles before failure; Frequency Domain Approach, and Miner's Rule etc. The evaluation of fatigue life under the application of random loading such as resonance was done previously with the Power Spectral Density functions within frequency domain and its validity was checked in time domain [4]. Palmgren Miner evaluated fatigue life using the ratio of load applied to the load allowable. In this research, Miner's rule is used because it is relatively simple, easy to apply, and deals with cumulative damage so the order of loading will not be important [5].

Usually theoretical analysis of fatigue life is based on the application of constant amplitude loading [6]. But in actual practice, there is always some kind of loading with variable amplitude acting on any structure like aircraft etc.

Ramesh Kumar and Balakrishnan designed a wing structure based on the static structural and modal analysis in ANSYS APDL whose results were further used to evaluate the fatigue characteristics and crack initiation in the material with the help of S-N curves. The fatigue life was estimated using the miner's rule [7].

II. RESEARCH METHODOLOGY

The specimen was designed and modeled in ANSYS and static structural analysis was performed based on which the maximum applicable stress was found. A variable amplitude loading spectrum (FALSTAFF) was generated and an algorithm was designed in MATLAB to calculate its number of cycles, and finally Miner's Rule was applied to the output of MATLAB algorithm and fatigue life was estimated.

A. Specimen

In this research, Aluminum Alloy 2024-T4 Compact Tension (CT) Specimen was used for analysis. CT specimen was chosen because it uses the least amount of material with given thickness and crack growth can be seen very easily. Its dimensions were calculated using ASTM Standard E-399[8]. Its geometry is shown in the Fig 1.

TABLE 1
AL-2024-T4 CHEMICAL PROPERTIES

Property	Value
Density	2.78 g/cc
Modulus of Elasticity	73.1 Gpa
Poison Ratio	0.33
Yield Strength	440 Mpa

B. ASTM Standard E399

ASTM (American Society for Testing & Materials) standard E399 is the guideline, according to which the specimen for fracture toughness testing and crack length was designed based on thickness. This standard is used for fracture toughness testing and it assumes plain strain conditions.

C. FALSTAFF

FALSTAFF is variable amplitude loading spectrum that closely approximates the variation of loading that an aircraft actually undergoes during its flight. The data for this spectrum was taken during the actual flight under the supervision of expert pilots. FALSTAFF is composed of 5 different flight conditions (like 1g, 2g, 4g etc) and at each flight condition, the variation in loads represents vibrations and hence the vibration at higher flight levels will be more critical because at that level, usually only a few cycles of stress will result in failure. The assumption in this spectrum is that each flight is of the duration of 5 hours and total 200 flights are present in one complete flight spectrum, which indicates 1000 flight hours [9]. One flight means that the aircraft starts from take-off condition, then climb, cruise, combat, again cruise and then land. These conditions were defined in terms of g's. Fig.2 shows one complete FALSTAFF spectrum in which different colors represent different flight condition [10].

Fig.2 has normalized loads on ordinate that made it possible to perform analysis for any type of loading either for Load factors, or stresses that an aircraft encounter during flight. For this the normalized value was multiplied by the desired maximum value and it converted all other normalized value according to that maximum stress value. The abscissa represents the number of load cycles. There were total 17983 stress cycles with different amplitudes present in one FALSTAFF spectrum. Different methods are available for calculating the number of cycles for variable loading like level count method, peak-trough method and rain-flow counting method. In this research, rain-flow counting method was selected because of the low computation power.

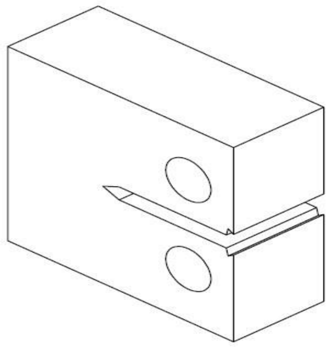

Fig. 1: Compact Tension Specimen

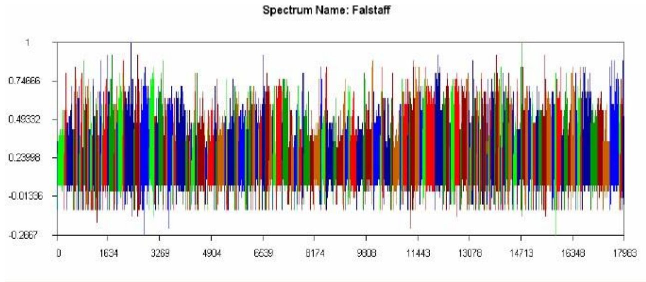

Fig. 2: FALSTAFF Spectrum

D. Rain-flow Counting Method

This method was used to calculate the number of stress cycles in variable amplitude loading spectrum. Three Important terminologies used in this method were

- Range: A line formed by joining two points. 'X' was taken to be the range under consideration. 'Y' was taken to be the previous range and 'S' was taken as the starting point.
- Peak & Valley: These were formed by joining recent three points under consideration [11]. An optimized MATLAB code was designed to implement this method. The input files for FALSTAFF was taken from AFGROW directory. AFGROW Software is made by US Air force and is used for crack growth simulations. Total number of stress cycles came out to be 17983 using designed program. Exceedance curve was also generated that represents the repetition of each of amplitude of stress. This curve was used to optimize the spectrum to apply it on any apparatus [12]. Exceedance curve is shown in Fig. 3.

For application of spectrum on apparatus, the high amplitude loads had to be filtered but for analytical analysis, the spectrum was applied without filtering. The analytical analysis was based on Miner's Rule which is explained below.

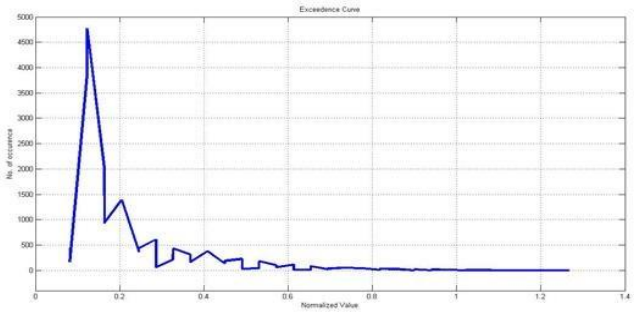

Fig. 3: Exceedance Curve

E. *Miner's Rule*

Miner's rule was used to find the cumulative damage caused by a certain number of load cycles. This rule works on the basis of the precession of the S-N curve as the number of stress cycles increases. Mathematically it can be written as

$$\sum_{i=1}^{N} \frac{D_i}{D} = 1$$

Where D_i represents the value of damage caused by the ith amplitude of load and D is the total damage [5]. This equation can also be written in another form like

$$\sum_{i=1}^{N} \frac{n_i}{N_i} = 1$$

Where n_i represents the number of cycles caused by the ith amplitude of load and N_i is the total number of cycles allowable corresponding to that amplitude of load [5]. The formula predicts the cumulative fatigue life, when the ratio of applied cycle to the total number of cycles become equal to the 1, this indicates that whole of the fatigue life of particular part has been utilized. So the basic requirement for every structure is that its value should always be less than 1.

III. COMPUTATION OF TEST DATA

There were two analysis involved in this research which were Static Stress Analysis and Fatigue Analysis.

A. *Static Stress Analysis*

This analysis was performed in ANSYS. Based on it, the region bearing maximum stress and its amplitude at crack was calculated. A series of steps were involved in static stress analysis; modelling of specimen, meshing, boundary conditions, loads and then stress analysis.

1) *Modelling of Specimen:* The specimen was designed using the standard E399 and is shown in Fig.4.

The same specimen was then modeled in CREO 2.0 and stress analysis was performed in ANSYS.

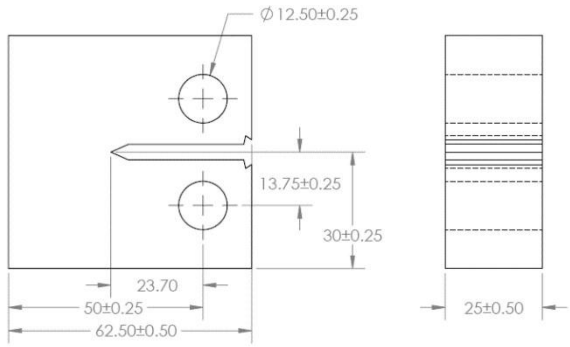

Fig. 4: Specimen Calculated Dimensions in mm

2) *Meshing:* Meshing was used to divide the whole specimen into chunks so that the computation is done at that segment. Smaller the length of mesh, more computational power will be required. First of all, the element width for the mesh was found by assuming the random values and error was observed. After the convergence, a mesh with an element width of 0.01mm at the crack tip was generated. Convergence means that there will not be any significant change in results even if the element width is further reduced. The detailed parameters of the mesh are shown in table 2. The Mesh is shown in the Fig.5.

TABLE 2
MESH PARAMETERS

Mesh Type	Triangular Free
Element Type	PLANE82
Element Width at Crack Tip	0.01mm
Element Width ahead of Crack Tip	0.05mm

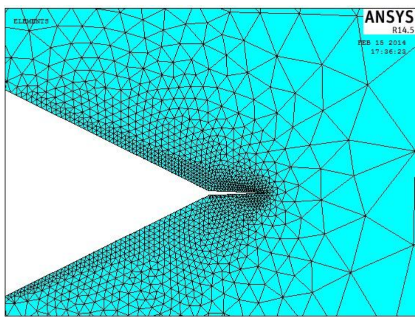

Fig. 5: Mesh at Crack Tip

3) *Boundary Conditions:* Equivalent loads were applied on both holes upper one and lower one

due to symmetry of the specimen, and fixed support boundary condition was applied on back side of specimen which specified the reference for it.

The purpose of this analysis was to find the maximum load that can be applied on the specimen. It should be equal to the yield strength or ultimate tensile strength of Al-2024. But the analysis was done on a notched specimen. The change in area caused stress concentration at notch. Also the stress was amplified to even greater value at the crack tip due to its small radius at the end of notch [13].

From the results of Von-Misses (Fig. 6), it can be seen that the maximum stress occurred at the crack tip. Hence crack tip was the most critical point for this research and all calculations were performed on that region.

Crack growth analysis cannot be performed in ANSYS because its working principle was based on Finite Element Method while the crack growth analysis requires Extended Finite Element Method so that the mesh keeps on updating at the crack tip only to save computational power. Hence it was calculated analytically.

To know the maximum value of stress, “Equivalent Von Misses Criteria” was used according to which the failure will occur when equivalent Von Misses Stress becomes equal to or greater than the yield stress [14]. The yield strength of Aluminum Alloy 2024-T4 was 440 MPa (Table 1). Different loads were applied and maximum load was then selected which met our failure criteria. As shown in table 3.

By using this loading criterion, maximum load that can be applied at the specimen calculated to be 32 MPa which amplified the maximum von misses load at notched region of high stress concentration to 432 MPa, which is the maximum applicable load before deformation.

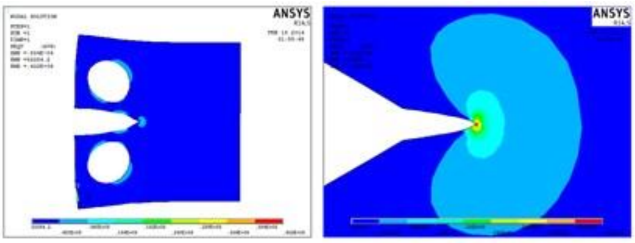

Fig. 6: Result-Von Misses

TABLE 3
DATA POINTS FOR DESIGNING MAXIMUM LOAD

Load Applied (Mpa)	**Stress concentrated at Notch Tip (Mpa)**
20	269
25	336
30	404
31	417
32	432
33	444
35	471
40	538
45	605
60	808

B. *Fatigue Analysis*

The Fatigue life was estimated using Miner’s Rule. The most important thing in the estimation using miner’s rule was to know that it will give only cumulative damage, not the sequential damage. First of all the maximum load for FALSTAFF spectrum was set to be 60% of maximum load found at the crack tip. Then a MATLAB code was generated based on which applied numbers of cycle of all amplitudes of loading were found. The algorithm of the program is shown in Fig.7.

The Algorithm shown is the implementation of Rain-flow counting method. After running this program, following results were obtained which indicates the total number of cycles applied corresponding to each amplitude of normalized load. Results are shown in Table 2.

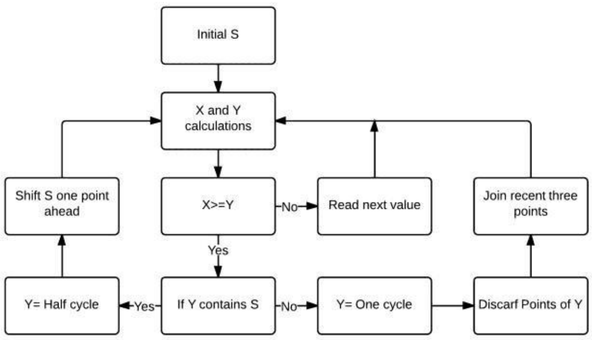

Fig. 7: Rain-flow Method Algorithm in MATLAB

TABLE 4
NORMALIZED LOAD CYCLES

Normalized Load	**No. of Cycles**
0.0817	444
0.0818	165

0.1225	3837
0.1226	4777.5
0.1634	2034
0.1635	933
0.2043	1388
0.2451	366
0.2452	432
0.286	611
0.2861	61
0.3268	214
0.3269	433
0.3677	311
0.3678	163
0.4086	375
0.4494	137
0.4495	185
0.4903	225
0.4904	32
0.5311	41
0.5312	180
0.572	102
0.5721	61
0.6129	113
0.613	8
0.6537	8
0.6538	85
0.6946	26
0.6947	35.5
0.7355	48
0.7764	38
0.8172	9
0.8173	28
TOTAL	**17983**

The data for loading and the number of load cycles corresponding to the load cycles were then compared against the S-N curve of Aluminum Alloy 2024-T4. The damage was calculated because of each load [15]. Every fraction of damage with respect to each cycle was calculated and was added to cumulative damage, based on which fatigue life was calculated.

IV. ANALYSIS OF TEST RESULTS

The cumulative damage to the material under observation was estimated by the help of Miner's rule for 45 different load steps which made the one spectrum of FALSTAFF so the formulae for total utilization of fatigue life became

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} \cdots\cdots\cdots \frac{n_{45}}{N_{45}} = 1$$

After the calculation of data taken from the program mentioned and the application of Miner's rule, the results were analyzed in three different terms which are shown below:

- By applying one spectrum, the cumulative damage was found to be 5.7 %, hence for failure 17.5 spectrums are required.
- The number of flights before failure were calculated by (200/0.057) which becomes 3499 flights.
- In term of flight hours the fatigue life was predicted to be 17500 flight hours.

V. CONCLUSION

As Al-2024 is used in aircraft structure so it is very important to overhaul the aircraft after certain intervals. These intervals were defined by the fatigue life calculated. This means that after almost 17500 hours, the aircraft's structure would fail. So, for safety, the aircraft should be conveyed to overhauling hanger just after 17250 hours. Further if some factor of safety has to be applied then these hours will be reduced to almost 17000 hours. This factor of safety will be set by Federal Aviation Requirements or Military Requirements.

In case of a commercial aircraft, the factor of safety should be very high because it involves hundreds of lives at risk. Hence the overhauling should be done just after 16000 hours to ensure safety and to compensate for the error of calculations.

By doing overhauling in time, there are certain advantages. First of all, the risk of death will be reduced due to less rate of fracture. Secondly if the structure remains in service for a longer time then the need to produce the new aircraft of the same type will be reduced, hence cost effective. Last but not the least, the reliability of the aircraft will be increased.

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