

# Development of an Orbit Propagator incorporating Perturbations for LEO Satellites

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*Abstract* — This paper deals with the development of orbit propagator for low Earth orbiting satellites incorporating dominant perturbations for them. This simulator involves modelling the space environment and the perturbing forces of interest and incorporating them in the two body Keplerian propagation to optimize the propagation. Orbit propagation algorithms and environment models have been developed and optimized using MATLAB-SIMULINK simulations for different scenarios. On SIMULINK, the models are developed using embedded blocks so that the model can be finally embedded into the DSP Kit. Space industry software are used for the code validation and the results obtained are found to be identical.

*Keywords* — Propagator, LEO, Perturbations, Embedded, DSP

## 1 INTRODUCTION

Orbit propagation is the process of using the dynamic equations of motion and models of environmental forces and torques to model the orbit for an extended period of time. Predicting the orbit of a satellite is an essential part of mission analysis and has impacts on the power system, attitude control and thermal design. It is the starting point in planning whether a proposed mission is feasible or not and how a satellite needs to be designed. The problem of reckoning the orbits of satellites, however, is not straightforward owing to a large number of gravitational and non gravitational factors affecting the simplistic two body motion. The main factors affecting the orbit of a satellite are: the non-spherical Earth, atmospheric drag, perturbed effects from the gravitational pull of the Sun and other planets, electromagnetic forces, radiation pressure and so on. These effects are of imperative concern for different types of satellite orbits. For example satellites in LEO are strongly affected by the non-spherical nature of the Earth and even atmospheric drag. Satellites out there in geostationary orbit, however, are amply far from the Earth for these effects to have any pronounced effect on their regular motion. The gravitational pull of the Sun and the Moon, nevertheless, does play a significant role in the evolution of the orbits at geo stationary altitude. Now the question arises for the need of an orbit propagator when there are a number of desktop propagators already available with the space trade e.g. Two Body, SGP4, and HPOP. Though for embedded applications, we necessitate a customized algorithm that may be used with on-board computer to propagate the orbit in real time.

## 2 SPECIFYING THE ORBIT

A perturbation is a deviation from some normal or expected motion. The perturbations need to be considered. Because the actual path will vary from theoretical two body path due to these perturbations. To understand the importance of incorporating these perturbations into an orbit propagator, we need to analyze the difference between the ideal orbit (a Keplerian orbit) and the real or evolved orbits.

### A. Keplerian Orbits

The two-body or Keplerian propagation is the simplest. It assumes that Earth is a perfectly symmetrical spherical mass & the only force is centripetal force resulting from satellite motion balancing the gravitational pull of the Earth and there are no other forces or perturbations influencing this ideal situation. So it's possible to propagate the mean orbital elements directly in time. Given the Keplerian elements for a single point in time, the estimation of the future position becomes relatively straight forward as the mean anomaly marches uniformly in time.

### B. Real Orbits

However we need to know that neither the Earth has a perfectly spherical symmetrical mass distribution nor gravity is the only force defining Earth-satellite motion. A body in space may have to face a number of Earth based or other influences depending upon its distance from the centre of the Earth. And thus the assumptions on which Kepler's propagation is based are not accurate. Certain improvements utilizing known irregularities can be made.

It should not be supposed that perturbations are always small. For example, ignoring the effect of the oblateness of the Earth on an artificial satellite would cause to completely fail in the prediction of its position over a long period of time. The purpose of this model is to provide means of propagating orbital elements to obtain position and velocity of a satellite incorporating those perturbations having noticeable effects on propagation of low Earth orbiting satellites.

### 3 PERTURBATIONS ON ORBITS AND ELEMENTAL VARIANCE

A Keplerian orbit is mathematical estimate at a particular time. A real orbit changes over time due to gravitational perturbations by other objects and atmospheric drag.

All Unperturbed, two-body orbits are always conic sections, so the Keplerian elements define an ellipse, parabola, or hyperbola. The real world is slightly more complex than the Keplerian model, and tracking programs recompense for this by introducing minor corrections to the Keplerian model. These corrections are identified as perturbations.

Evolution of the orbital elements in low Earth orbiting satellites takes place mainly due to the gravitational pull of bodies other than the primary and the non spherical shape of the primary as well as atmospheric drag, the perturbations considered in the above propagator.

There are two types of perturbations. *Gravitational perturbations* include the geo potential perturbing acceleration, solid Earth tide effect, ocean tide effect, rotational deformation of the Earth, third body effect, and relativity effects especially general relativity for low Earth orbiting satellites. *Non gravitational perturbations* are comprised of atmospheric drag, solar radiation pressure and tidal friction.

#### 3.1 Effects of the Non-Spherical Earth

The Non-Spherical Earth results in the progression of the orbit and thus causes the right ascension of ascending node (RAAN) and argument of perigee  $\omega$  to rotate at the rates of several degrees per day.

Mathematically, this effect is modelled by expanding the geo-potential function  $U_{\phi}^r \theta$  at point  $(r, \theta, \phi)$  in series of spherical harmonics as follows:

$$U_{\phi}^r \theta = \sum_{n=0}^{\infty} \left(\frac{r}{R}\right)^{n+1} J_n P_{n0} \cos \theta + \sum_{n=1}^{\infty} \sum_{m=1}^n \left(\frac{r}{R}\right)^{n+1} [C_{nm} \cos m\phi + S_{nm} \sin m\phi] P_{nm} \cos \theta \quad [1]$$

Here,  $J_n$  are zonal harmonic coefficients,  $P_{nm}^{[2]}$  are Legendre polynomial and  $C_{nm}$  and  $S_{nm}$  are tesseral<sup>[4]</sup> harmonic coefficients for  $n \neq m$  and sectoral<sup>[4]</sup> harmonic coefficients for  $n = m$ .

This model is a set of coefficients in the spherical harmonic expansion. These models are obtained using tracking data for a large number of satellites. A  $21 \times 21$  model means that it is a  $21 \times 21$  spherical harmonic expansion. The need for higher accuracies has led to more complex geo potential models; although most of the terms in higher order models will be extremely small.

GEMT geo-potential model is used for this purpose, which is a  $36 \times 36$  model.

A higher order model is considered. This is because although the magnitude of higher order harmonics is small, they have a significant effect in special circumstances. E.g. in a repeating ground (satellite flies over the same locations on the ground, day after day), resonances may build up when the satellite continuously passes, such that the effects of tesseral harmonics becomes more important.

#### 3.2 Third Body Interaction

In general, addition of solar and lunar body gravity contributions for Earth-orbiting satellites is sufficient for accuracy in all but the most demanding applications.

The gravitational forces of the Sun and the Moon cause periodic variations in all the orbital elements. This effect is akin to effect of Earth equatorial bulge i.e. both the Sun and the Moon apply the external torque to the orbits. In LEOs the consequence is dominated by the Earth rotations caused by the Earth's oblations.

Mathematically, the perturbation of a third body is expressed as gravitational acceleration,  $\mathbf{a}_{\text{planet}}$ , exerted by the planet on the satellite minus the acceleration exerted on the Earth by the same body.

$$\frac{d^2 \mathbf{r}}{dt^2} + \frac{\mu \mathbf{r}}{|\mathbf{r}|^3} = \mathbf{a}_{\text{planet}} \quad [3]$$

Here  $\mu$  is the Gravitational parameter of the third body and  $\mathbf{r}$  is the vector from its center to the satellite. The position vector to the third bodies (e.g. Sun, Moon) were calculated using Julian date and space craft vector in ECI frame.

#### 3.3 Atmospheric Drag and Orbital Decay

It is the principal non-gravitational force acting on low Earth orbiting satellites. It acts opposite to velocity vector and thus removing energy from the satellite orbit. This reduction in energy causes the orbit to get smaller leading to a further increase in the drag. When this happens the body will rapidly spiral down and intersect the central body unless external energy is provided to help it stay in orbit.

The acceleration due to atmospheric density depends upon the height above Earth's surface, drag coefficient and cross-sectional area of the satellite face perpendicular to velocity vector. The drag acceleration  $\mathbf{a}_{\text{drag}}^{[4]}$  on a space craft moving with velocity  $\mathbf{v}$  is:

$$\mathbf{a}_{\text{drag}} = -0.5 * \rho * \mathbf{v}^2 * \mathbf{v}_{\text{Aero}} * CD * A/m \quad [4]$$

Here,  $\rho$  is the atmospheric density at altitude of the satellite,  $CD^{[4]}$  is the drag coefficient,  $A$  is the area and  $m$  is the mass of the satellite. For computing the atmospheric density, 1976 Standard<sup>[4]</sup> model is used. It is a table look-up model based on the satellite's altitude, with a valid range of 86km - 1000 km.

### 4 HIGH FIDELITY ORBIT SIMULATOR

This is a high fidelity orbit simulator incorporating all the perturbations as does the STK's HPOP propagator. The complete implementation of the said propagator includes modelling environmental effects on two-body propagation using MATLAB and then, converting into Simulink Model. The program flow is diagrammatically shown below.

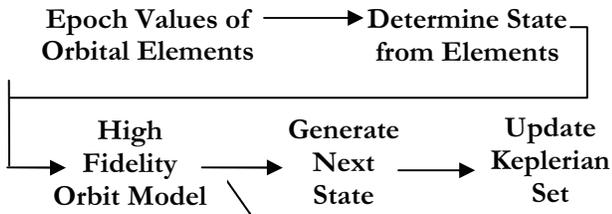


Figure 1 - Customized Orbit Propagator Flow Diagram

A. Customized HPOP Inputs/Outputs Specifications

The propagator calls for initial position and velocity components in ECI (Earth Centred Inertial) coordinate system. Initial position and velocity vectors can be input directly from GPS data or the values of orbital elements at epoch is used for determining the state and initiating the propagation.

Moreover, orbit propagation tolerance, external disturbances, acceleration vectors and optional values for propagation (the gravitational coefficients, the drag coefficient etc.) can be easily changed depending upon the requirement. Next, the orbit is propagated. The outputs of propagation are the updated Keplerian elements evolved due to the propagation with time and with the perturbing accelerations as well.

B. Implementation on SIMULINK

Only Embedded MATLAB Function blocks are used. The MATLAB function executes for simulation and generates code for a Real-Time Workshop® target. The Embedded MATLAB Function block supports a subset of the language for which it can generate efficient embeddable code so that the model may be directly embedded on DSP kit.

C. Individual Blocks

Three embedded MATLAB-SIMULINK blocks are used. The purpose of each of them is illustrated below. In addition, the subsystem labelled “Feedback” is used to help propagation based on the most recent state available with the satellite.

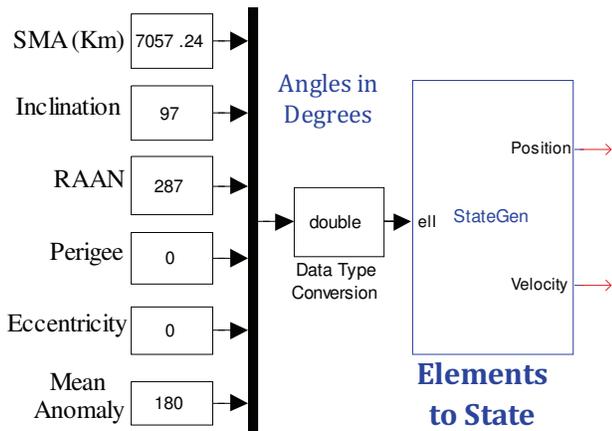


Figure 2 - State Generation SIMULINK Block

1) StateGen SIMULINK Block

The first embedded block **StateGen** takes epoch values of Keplerian elements as inputs and employs them for initial state generation<sup>[7]</sup> (ECI position and velocity vectors). Here true

anomaly is the time stamp). True anomaly is determined from mean anomaly using the Kepler’s equation<sup>[4]</sup>.

2) OrbGen SIMULINK Block

Embedded block **OrbGen** is the main block. It contains Fourth/fifth order Runge-Kutta<sup>[5]</sup> integrator and environment models requisite for accurate prediction of orbit.

Position and velocity vectors are inputted from StateGen block. Integrator needs tolerance on error and up to eight optional arguments (Options, Julian date, external disturbances acceleration, gravitational parameter and radius of the Earth, and spherical harmonics series). See figure 3.

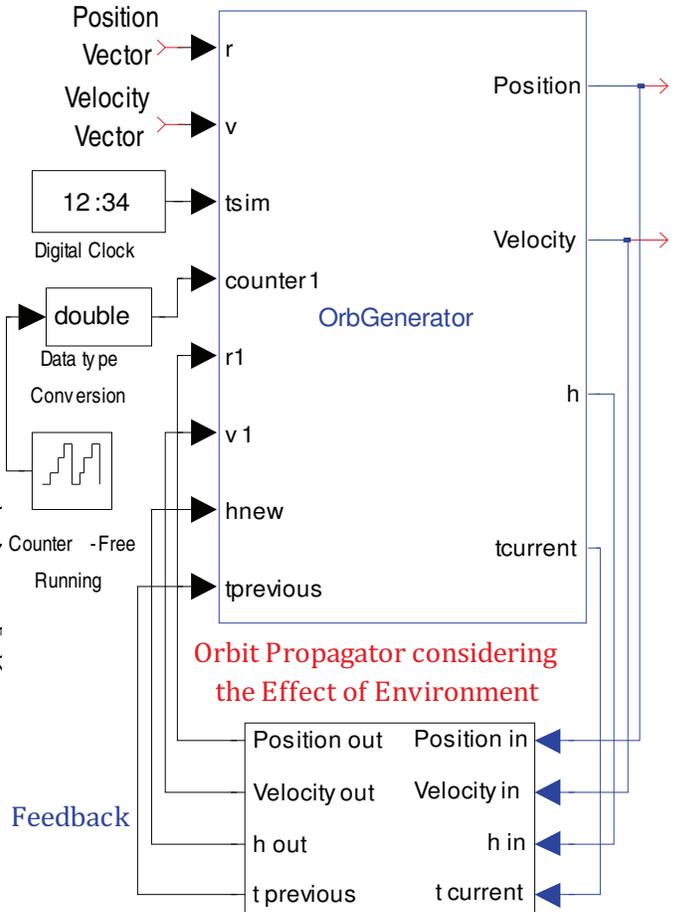
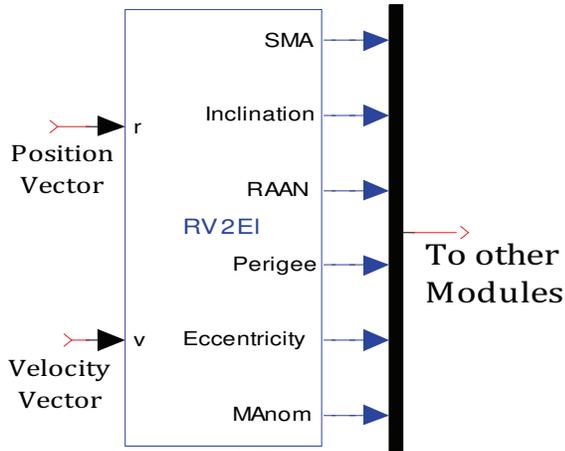


Figure 3 - OrbGen SIMULINK Block

3) RV2EL SIMULINK Block

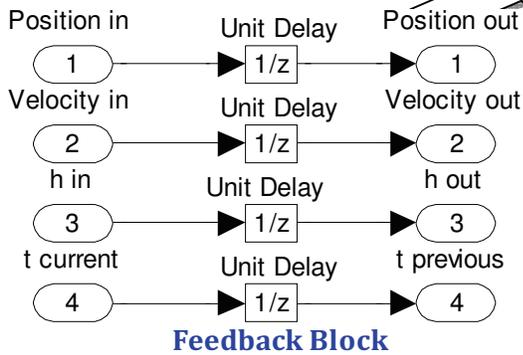
The last embedded block is **RV2EL** which converts evolved state into Keplerian elements set. See figure 4.



**Figure 4 - State to Keplerian Elements SIMULINK Block**

4) *Feedback SIMULINK Block*

Feedback is used with *OrbGenerator* to drive next state from current state. See figure 5.



**Figure 5 - Feedback Block**

D. *Results Comparison with STK*

Simulations are performed on MATLAB and STK<sup>[7]</sup> with epoch orbital elements. The x axis shows time in hours. Title shows the element for which error is shown. Error is calculated minus simulated value. The results shown are for 3 days. As the plots show, the prediction is very accurate and errors are very low.

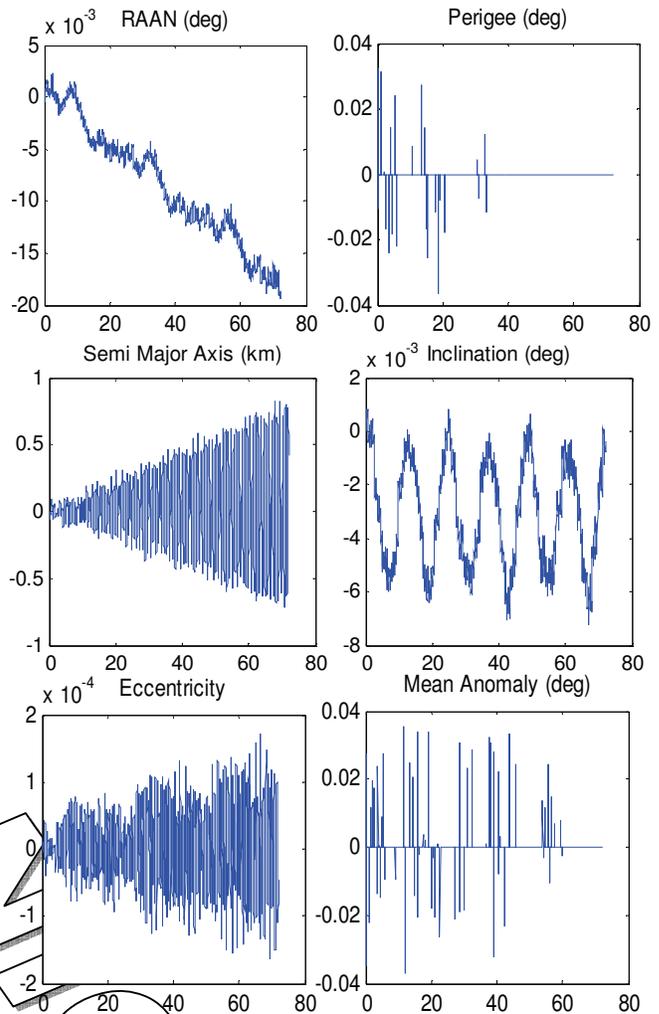
In general, the smaller the time step, the better the prediction. The propagator works on SIMULINK clock.

Propagator inputs should be regularly updated using GPS data to counter accumulated propagation errors.

**5 OPTIMIZATION**

When working on SIMULINK, following issues were addressed. First, since some MATLAB functions aren't supported in embedded MATLAB function block, these functions were rewritten to imitate their functions. Second, to use memory efficiently, data was utilized at the cost of efficiency and accuracy. For this, variables were type casted from double, which is default, to single. Third, to give the output of the block to its own input is an illegitimate practice

in MATLAB, so the delay blocks were added with some educate initial conditions.



**Figure 6 - Error Plots in Keplerian Elements Set between STK and Customized Orbit Propagator**

To accomplish the real time mission needs, SIMULINK based propagator model we need to embed into some hardware, once the complete model is successfully tested in MATLAB® SIMULINK environment. As an example, TMS3206713 DSK from Texas Instruments™ can be used to achieve the aim. That is why the code was converted into embeddable form.

**6 CONCLUSIONS**

The goal of the task was to develop a customized propagator addressing the mission needs of a LEO satellite considering all the perturbations influencing its propagation. Results are evaluated against STK simulations and are found to be matched satisfactorily. A number of perturbations may be included at a time to see how the individual perturbations affect the satellite propagation when it is in its orbit and to see the improvements in propagation if those perturbations are taken into account.

To conclude we can say that the code provides high accuracy in predicting the orbit and may be embedded into the onboard hardware for real time orbit propagation. The updated state may be used by other modules if needed. This satisfies the mission needs of the low Earth orbiting satellites.

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